STAT 302 – Introduction to Probability

Assignment 1 – DUE: 3 October 2025

Problem	Points	max.
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		100

SURNAME First Name

Signature Student ID

Problem 1 [10 points]

Suppose we repeatedly roll two fair six-sided dice, and look at the sum of the two values. What is the probability that the first time the sum is exactly 7 is on the 3rd roll?

Problem 2 [10 points]

An instructor gives her class a set of 10 problems with the information that the final exam will consist of a random selection of 7 of them. If a student has figured out how to do 7 of the problems, what is the probability that they

(a) will answer all 7 problems correctly?

(b) will answer at least 5 of the problems correctly?

Problem 3 [10 points]

Suppose we roll a fair six-sided die, and flip 6 coins. What is the probability that the number of heads equals the number on the die?

Problem 4 [10 points]

There are two urns (# 1 and # 2). Urn # 1 has 5 red and 7 blue balls, while urn # 2 has 6 red and 12 blue balls. We pick three balls at random from each of the two urns (without replacement). What is the probability that all 6 balls are of the same colour?

Problem 5 [10 points]

Suppose there is a 40% chance that your team wins or ties today's game, and that there is a 30% chance that they win. What is the probability that they tie?

Problem 6 [10 points]

Consider a standard set of poker cards (52 cards, 13 of each of four suits (diamonds, hearts, spades and clubs)). The cards of each suit are numbered 2 to 10, J, Q, K and A. A hand consists of 5 randomly chosen cards. A "full house" consists of a hand with two cards with the same face value, and the other three cards also sharing a face value. For example, $(8, 2, 8, 2, 2)$ is a "full house", but $(Q, 9, Q, Q, Q)$ is not. If all hands are equally likely, what is the probability of receiving a "full house"?

Problem 7 [10 points]

A set of N people, $(N \geq 3)$, including persons A, B and C, are randomly arranged in a line.

(a) What is the probability that A, B and C sit together? (in other words, that A sits next to B or C, and B sits next to A or C, and C sits next to A or B).

(b) What would the probability be if the people were randomly arranged in a circle?

Problem 8 [10 points]

How many different arrangements of the letters in the word "volcanologists" can be obtained?

Problem 9 [10 points]

For the following claims, state whether they are TRUE or FALSE. Statements claimed to be TRUE must be accompanied by a proof, and statements claimed to be FALSE must be accompanied by a counterexample.

NOTE: For a statement to be TRUE, it **MUST** be TRUE **for any possible** relevant events for the statement in question. For example, for (a) below to be TRUE, it must be true **for any arbitrary events** A and B, not only for independent ones.

(a) Let A and B be events in the sample space Ω. Then $P(A \cap B) \leq P(A)P(B)$.

(b) Let E be an event with $0 < P(E) < 1$ and define $P_E(A) = P(A \cap E)$ for every event A in the sample space S. Then P_E satisfies the 3 axioms of probability.

(c) Let A and B be events in the sample space Ω. Then $P(A \cap B^C) = P(A \cup B) - P(B)$.

(d) Let $A_1, A_2, \ldots, A_n$ be events in the sample space Ω. Then

$$P\left(\cap_{i=1}^{n} A_i\right) \leq \min_i \{P(A_i)\}.$$

Problem 10 [10 points]

Suppose you want to divide a 52 card deck into four hands with 13 cards each.

(a) What is the probability that each hand has a queen?

(b) What is the probability exactly two of them have a queen?