

STAT 302 – Introduction to Probability
Assignment 2 – DUE: 24 October 2025

SURNAME	First Name
Signature	Student ID

Problem	Points	max.
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		100

Problem 1 [10 points]

Assume that the probability of a traffic accident occurring at a certain intersection is 10%. At the same time, the probability that it snows is 20%. Furthermore, suppose that the conditional probability of having an accident, conditional of the event that it snows, is 40%. Compute the conditional probability that it snows given that there is an accident at that intersection.

Problem 2 [10 points]

We roll two fair, 6-sided dice. One of the dice is red, the other one is blue. Let A be the event that the 2 dice show the same face value, B the event that the sum of the two dice is 12, C the event that the red die shows 4, and D that the blue die shows 4.

- (a) Prove whether A and B are independent or not.
- (b) Prove whether A and C are independent or not.
- (c) Prove whether A and D are independent or not.
- (d) Prove whether A , C , and D are independent or not.

Problem 3 [10 points]

We first roll a fair 6-sided die. Next, we toss a number of fair coins equal to the number showing on the die (e.g., if you roll a 5, then you toss 5 coins). Calculate the conditional probability that the number of heads is 3 given that the die roll is a 5.

Problem 4 [10 points]

We have 3 2-sided cards. One is red on both sides, one is blue on both sides, and the third one has one side of each colour. The cards are put in a hat, and one is picked at random and placed on the table (each side is equally likely to be showing). Calculate the conditional probability that the card is the one with 2 red sides, given that the face showing is red.

Problem 5 [10 points]

Let A and B be two arbitrary events with $P(A \cup B) = 1$. Prove that A and B are independent if and only if $\max\{P(A), P(B)\} = 1$.

Problem 6 [10 points]

We draw 5 cards at random from a standard poker deck with 52 cards (as described in Assignment 1, for example). Label their face values as 1, 2, ..., 13. Let Y be the smallest face value on the 5-card draw. Calculate $E(Y)$, using a computer if needed (in which case you need to include your code in your submitted answer). Note: you should not estimate $E(Y)$, you need to compute its exact value (up to your computer's numerical precision).

Problem 7 [10 points]

We toss 2 coins. Let $X = 1$ if the first coin is "heads", and $X = 0$ otherwise. Let $Y = 1$ if the second coin is "heads", and $Y = 5$ otherwise. Let $Z = XY$. Find the CDF of Z .

Problem 8 [10 points]

There is a loop in a computer program, which exits with probability $1/3$, every time it runs. What is the probability that the loop runs 5 times before exiting?

Problem 9 [10 points]

Let X be a random variable with $\mathcal{R}_X = \mathbb{N} = \{1, 2, \dots\}$. Its PMF is $f_X(a) = 2^{-a}$ for $a \in \mathcal{R}_X$, and $f_X(a) = 0$ if $a \notin \mathcal{R}_X$. Calculate $V(X)$. Hint: the distribution of $Z = X - 1$ belongs to one of the families discussed in class.

Problem 10 [10 points]

Let X be the value obtained by the roll of a fair 6-sided die, and let $Y = |X - 3|$.

- (a) Find the CDF of Y .
- (b) Find $E(Y)$ and $V(Y)$.