

STAT 302 – Introduction to Probability
Assignment 3 – DUE: 14 November 2025

SURNAME	First Name
Signature	Student ID

Problem	Points	max.
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		100

Problem 1 [10 points]

On a specific day, a car salesperson has scheduled 2 sales appointments. His first appointments lead to a sale with probability 0.3, and second ones lead independently to a sale with probability 0.6. Any sale made is equally likely to be either for a high-end car with a \$120,000 sale price, or an economy car which costs \$35,000. Let X be the dollar amount of the salesperson total sales for this day. Compute $\mathbb{E}(X)$.

Problem 2 [10 points]

Suppose there are 2 urns. Urn number 1 contains 100 chips: 30 are labelled 1's, 40 are labelled 2's, and 30 are labelled 3's. Urn number 2 contains 100 chips: 20 are labelled 1's, 50 are labelled 2's, and 30 are labelled 3's. A coin is tossed and if a head is observed then a chip is randomly drawn from Urn 1, otherwise a chip is drawn from Urn 2. Let Y be the value of the drawn chip. If the occurrence of a head on the coin is denoted by $X = 1$, a tail by $X = 0$, and $X \sim \text{Bi}(1, 3/4)$, find:

- (a) $\mathbb{E}(X|Y)$
- (b) $\mathbb{E}(Y|X)$
- (c) $\mathbb{E}(Y)$
- (d) $\mathbb{E}(X)$

Problem 3 [10 points]

The monthly worldwide average number of nuclear plant critical incidents is 3.5. Let X be the number of upcoming consecutive incident-free quarters, starting in 2026. There are 4 quarters in a year. In other words, X is the number of incident-free quarters observed in a row, starting January 1, 2026, say. For example, if the next incident occurs after 10 months, then $X = 3$. If $X = 2$, there were no incidents in the first 6 months, and at least one incident between months 6 and 9. What is $\mathbb{E}(X)$?

Problem 4 [10 points]

Consider a box containing the 13 diamond cards from a regular deck of cards. Assume, for simplicity, that they are numbered 1, 2, . . . , 13. We randomly draw 3 cards without replacement. Let X be the lowest card face (number) in the draw, and Y be the highest card face in the draw. Prove whether X and Y are independent.

Problem 5 [10 points]

Let $X \sim \mathcal{P}(\lambda)$ for some $\lambda > 0$. We have $Y|X = k \sim \text{Bi}(k, p)$, where $p \in (0, 1)$. Prove that $X|Y = 0 \sim \mathcal{P}(\lambda(1 - p))$.

Problem 6 [10 points]

Find:

- (a) $\mathbb{E}[(X - 2)^2]$, if $\mathbb{E}(X) = 0$ and $\text{Var}(X) = 2$.
- (b) $\text{Var}(5X^2)$, if $\mathcal{R}_X = \{-1, 0, 1\}$, $\mathbb{E}(X) = 0$, and $\text{Var}(X) = 1/2$.
- (c) $V(a^Y)$, if $a > 0$ and $Y \sim \mathcal{P}(\lambda)$, for some $\lambda > 0$.

Problem 7 [10 points]

We roll a fair 6-sided die. Let X be its observed value. We then we toss X fair coins, and let Y be the number of heads obtained. Using a computer if necessary, find $\mathbb{E}(Y)$ and $\mathbb{E}(X|Y = 0)$.

Problem 8 [10 points]

Let X be a discrete random variable with $f_X(-1) = f_X(1) = 1/4$ and $f_X(0) = 1/2$. Let $Y = X^2$. Show that $\text{cov}(X, Y) = 0$ but X and Y are not independent.

Problem 9 [10 points]

Consider a discrete random vector (X_1, X_2) with point mass probability function given by

		X_2		
		0	1	2
X_1	-1	1/6	0	1/6
	0	0	a	a
	1	1/6	1/6	0

for some $a \in [0, 1]$. Note that $\mathcal{R}_{X_1} = \{-1, 0, 1\}$ and $\mathcal{R}_{X_2} = \{0, 1, 2\}$.

- (a) If $V(X_2) = 2/3$, compute $V(X_1 - 2X_2)$.
- (b) Compute $E(X_1|X_2)$.

Problem 10 [10 points]

Let X and Y be two random variables, not necessarily independent, such that $X \sim \text{Bi}(1, p)$ and $Y \sim \text{Bi}(1, q)$, where $p, q \in (0, 1)$. Let $Z = X + Y$ and $W = X - Y$. Show that if $p + q = 1$ then $\text{cov}(Z, W) = 0$, and check whether Z and W are also independent or not.

Note: Such non-independent X and Y can be constructed as follows. Let $X \sim \text{Bi}(1, p)$, and let Y be a function of X as follows: if $X = 1$, then $Y = 1$; if $X = 0$ then we toss a biased coin with $\mathbb{P}(\text{Heads}) = \alpha$, and let $Y = 1$ if you observe heads, and $Y = 0$ otherwise. Then Y has a Binomial distribution, and if we choose $\alpha = (1 - 2p)/(1 - p)$, then $\mathbb{P}(X = 1) + \mathbb{P}(Y = 1) = 1$, and X and Y are not independent.