

STAT 302 – Introduction to Probability
Assignment 4 – DUE: 28 November 2025

SURNAME	First Name
Signature	Student ID

Problem	Points	max.
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		100

Problem 1

The time it takes to connect a caller with a customer service representative (in hours) is an exponential random variable with expected value λ .

- (a) [3 points] If $\lambda = 3$, what is the probability that the next time you call you will need to wait less than one hour to be connected with a person ?
- (b) [7 points] The number of people calling the call centre in any one hour period follows a $\mathcal{P}(10)$ distribution. What should $\lambda > 0$ be so that the probability that none of the customers calling in the next hour has to wait more than hour to be connected, is at least 95%?

Problem 2

Let $Y = e^{-X}$ where $X \sim \mathcal{E}(\lambda)$, for some $\lambda > 0$.

- (a) Find the value of λ the maximizes $V(Y)$
- (b) What is $\lim_{\lambda \rightarrow \infty} V(Y)$?

Problem 3

Let (X, Y) have joint pdf

$$f_{X,Y}(x, y) = \begin{cases} 8xy & \text{if } 0 \leq x \leq a, x \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find a so that the above is a valid pdf. Is it unique?
- (b) Show that $\mathbb{E}(Y|X = x)$ is an increasing function of x .

Problem 4

Let $X \sim \mathcal{E}(\lambda)$, and $Y|X = x \sim \mathcal{U}(0, x)$.

- (a) Find $\mathbb{E}[Y]$.
- (b) Using a computer if necessary, find $\mathbb{P}(Y \geq 1)$ when $\lambda = 2$.

Problem 5

Let X and Y be continuous random variables with joint pdf

$$f_{X,Y}(x, y) = e^{-x}, \quad 0 < y < x < \infty$$

What is $\text{Corr}(X, Y)$?

Problem 6

Let $X \sim \text{Uniform}(0, 1)$, and $Y|X = x \sim \mathcal{E}(1/x)$.

- (a) Find $\mathbb{E}[X|Y = y]$.
- (b) Find $V(Y)$.

Problem 7

Let $X \sim \mathcal{U}(0, 1)$.

- (a) Show that for any fixed $a \in (0, 1)$, $h(k) = \mathbb{P}(X^{1/k} \leq a)$ is a decreasing function of $k \in \mathbb{N}$.
- (b) Find k_0 such that for any $k \geq k_0$ we have $\mathbb{P}(X^{1/k} > 3/4) > 3/4$.

Problem 8

Let X, Y be two random variables such that their joint PDF is given by $f_{(X,Y)}(x, y) = g(y - a - bx)f_X(x)$ for $x, y \in \mathbb{R}$, where a and b are fixed real numbers, $g : \mathbb{R} \rightarrow \mathbb{R}$ is a function that satisfies $\int_{-\infty}^{+\infty} g(t) dt = 1$ and $\int_{-\infty}^{+\infty} t g(t) dt = 0$, and f_X denotes the PDF of X . Show that $\mathbb{E}[Y|X] = a + bX$.

Problem 9

Let X and Y be independent random variables, each uniformly distributed on $[0, 1]$. Find $\mathbb{P}(\min\{2X + Y, X + 2Y\} \leq 1)$.

Problem 10

Let (X, Y) have joint pdf

$$f_{(X,Y)}(x, y) = \begin{cases} 2, & 0 < x < y < 1, \\ 0, & \text{otherwise.} \end{cases}$$

Find $\mathbb{P}(Y - 2X < 0)$.