STAT 302: Assignment 1 Solutions

Summer 2026, Due May 20th, 11:59pm on Gradescope.

Question 1 [6 points]

Suppose that a student's phone screentime report shows that they are scrolling Reddit 50% of days, checking Instagram 80% of days, and checking their email 10% of days. 30% of the days, the student checks Reddit and Instagram in the same day (we don't know whether or not they also check their email on these days). 17% of the days, they exclusively look at Reddit. They never check their email without opening Reddit or Instagram. 3% of the days they check all three apps.

Show your work and all calculations. You may use visuals to help you.

(a) On a randomly selected day, what is the probability that the student checks only Reddit and their email?

(b) On a randomly selected day, what is the probability that the student only checks Instagram, but not email nor Reddit?

(c) What is the probability that they do not check any of the three apps?

Solution

(a) 3%

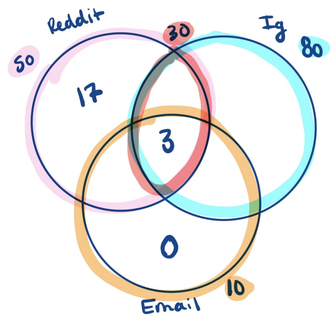
(b) 46%

(c) 0%

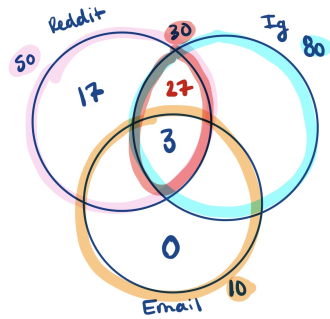
You can fill out a Venn Diagram to easily solve for each piece:

For each part:

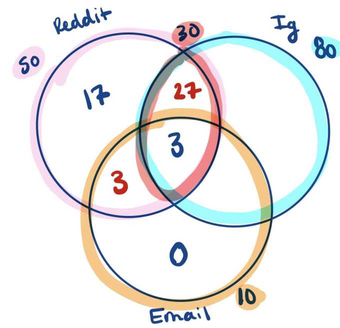
- 1 point for correct answer
- 1 point for showing work



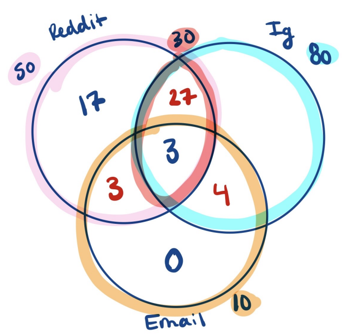
Step 1



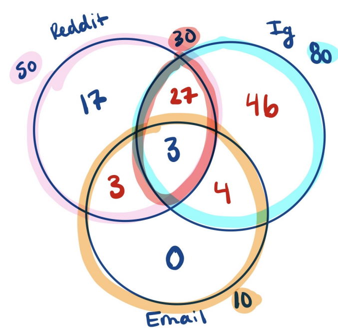
Step 2



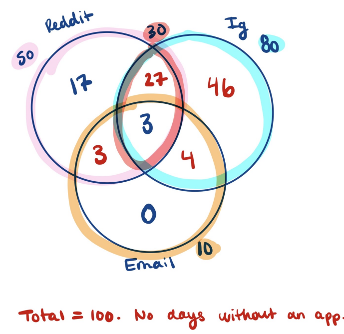
Step 3



Step 4



Step 5



Step 6

Question 2 [8 points]

Prove the Bonferroni inequality:

$$\mathbb{P}\left(\bigcap_{i=1}^{n} A_i\right) \geq \sum_{i=1}^{n} \mathbb{P}\left(A_i\right) - (n-1)$$

Show every step of the proof. List any results, rules, or theorems you use in the respective lines of your proof.

Hint:

$$\bigcap_{i=1}^{n} A_i = \left(\bigcup_{i=1}^{n} A_i^c\right)^c.$$

Solution

$$
\begin{aligned}
\mathbb{P}\left(\bigcap_{i=1}^{n} A_i\right) &= \mathbb{P}\left(\left(\bigcup_{i=1}^{n} A_i^c\right)^c\right) \\
&= 1 - \mathbb{P}\left(\bigcup_{i=1}^{n} A_i^c\right) && \text{complement rule} \\
&\geq 1 - \sum_{i=1}^{n} \mathbb{P}\left(A_i^c\right) && \text{Boole's inequality} \\
&= 1 - \sum_{i=1}^{n} \left(1 - \mathbb{P}\left(A_i\right)\right) && \text{complement rule} \\
&= 1 - \sum_{i=1}^{n} 1 + \sum_{i=1}^{n} \mathbb{P}\left(A_i\right) \\
&= 1 - n + \sum_{i=1}^{n} \mathbb{P}\left(A_i\right) \\
&= \sum_{i=1}^{n} \mathbb{P}\left(A_i\right) - (n-1)\,.
\end{aligned}
$$

- 1 point for each of the 6 starred lines of the proof.
- 1 mark for naming Boole's inequality.
- 1 mark for appropriately using the $\geq$ symbol (exactly once) in the proof.

Question 3 [7 points]

Prove that $\mathbb{P}(\cdot \mid B)$ is a probability. Hint: there are three axioms in the formal definition of a probability.

Solution

We must show from the definition that this function satisfies the 3 axioms.

Axiom 1: $\mathbb{Q}_B\left(\Omega\right) = 1$:

$$\mathbb{Q}_B\left(\Omega\right) = \frac{\mathbb{P}\left(\Omega \cap B\right)}{\mathbb{P}\left(B\right)} = \frac{\mathbb{P}\left(B\right)}{\mathbb{P}\left(B\right)} = 1$$

Axiom 2: $\mathbb{Q}_B\left(A\right) \geq 0$ for any $A \in \Omega$:

$$\mathbb{Q}_B\left(A\right) = \frac{\mathbb{P}\left(A \cap B\right)}{\mathbb{P}\left(B\right)} \geq 0$$

Axiom 3: If $\{A_i\}_{i\geq 1}$ are disjoint events:

$$\begin{aligned}
\mathbb{Q}_B\left(\bigcup_{i=1}^{\infty} A_i\right) &= \mathbb{P}\left(\left[\bigcup_{i=1}^{\infty} A_i\right] \bigcap B\right) \Big/ \mathbb{P}(B) \\
&= \mathbb{P}\left(\bigcup_{i=1}^{\infty} [A_i \cap B]\right) \Big/ \mathbb{P}(B) \\
&= \left[\sum_{i=1}^{\infty} \mathbb{P}(A_i \cap B)\right] \Big/ \mathbb{P}(B) \\
&= \sum_{i=1}^{\infty} \frac{\mathbb{P}(A_i \cap B)}{\mathbb{P}(B)} \\
&= \sum_{i=1}^{\infty} \mathbb{Q}_B(A_i)
\end{aligned}$$

- 1 mark identifying Axiom 1
- 1 mark for showing Axiom 1 holds
- 1 mark for identifying Axiom 2
- 1 mark for showing Axiom 2 holds
- 0.5 marks for identifying Axiom 3
- 0.5 marks for each line of the proof.
- No deductions if they list the fourth axiom about the empty set, but we don't need to show it here.

Question 4 [8 points]

A quality control inspector examines items coming off a production line and classifies each as defective (D) or non-defective (N). The inspector examines items one at a time and **stops as soon as they find 2 defective items, or after examining 4 items**, whichever comes first.

(a) Write out the sample space Ω.

(b) Let A be the event that the inspector stops after exactly 3 items. List the outcomes in A

(c) Let B be the event that fewer than 2 defective items are found in total. List the outcomes in B.

(d) What is $A \cap B$? Describe this subset in words.

Solution

(a) [2 points]

$\Omega = \{DD, NDD, DND, NNDD, NDND, DNND, NNNN, DNNN, NDNN, NNDN, NNND\}$

- 0.25 point deduction for each missing element in the sample space, up to a maximum of 2 points

(b) [2 points]

If the inspector stops after three items, it means they found two defects, and these two defects were not the first two. $A = \{NDD, DND\}$.

- 1 point per element in the sample space.

(c) [2 points]

If there are fewer than two defective items in total, then that means there are either 0 or 1 defectives across the whole sequence. And that means we'd need four items.

$B = \{NNNN, DNNN, NDNN, NNDN, NNND\}$

- 0.5 point deduction per missing element in B, up to a maximum or 2.

(d) [2 points]

Sequences where the inspector stopped after three items and there were fewer than 2 defects. This isn't possible as an inspector will only stop at 3 items if there are 2 defects. Therefore, $A \cap B = \emptyset$

- 1 point for description
- 1 point for identifying it's empty.