STAT 302: Assignment 2

Summer 2026, Due May 27th, 11:59pm on Gradescope.

Question 1 [12 points]

A University is developing an AI-driven plagiarism detector. The detector returns positive (says a student plagiarized) 92% of the time for students who did plagiarize.

Assume 97% of the student body did not plagiarize on an assignment. The probability of testing negative for plagiarism across all students is 85.6%.

(a) **[1.5 points]** Let C be the event that a student plagiarized. Let T be the event that the detector returns positive (says a student plagiarized). Write the quantities that are given in this question as probabilities, in terms of these events.

(b) **[1.5 points]** What is the probability of testing negative, given that you did not plagiarize? Show all steps of this calculation.

(c) **[2 points]**A student is flagged by the detector. What is the probability that they actually plagiarized?

(d) **[3 points]**The University is considering a two-stage screening process. Students who test positive have their assessments run through the detector again. The results of the second test used to flag whether or not the student plagiarized. What is the probability that, even with this strategy, the detector is incorrect? Assume each the two tests are conditionally independent, given the plagiarism status. Use the notation T_1 and T_2 to denote the respective tests.

e) **[3 points]** Using the same two-stage screening process as in (d), what is the probability that a student didn't plagiarize, given they were flagged?

f) **[1 point]**Based on your answers for (d) and (e), would you be comfortable if your course implemented this two-stage screening process? Answer in 1 - 2 sentences.

Question 2 [9 points]

A student union runs a raffle to raise money for campus events. Each ticket costs **$3** to enter. There are **500 tickets** sold in total, and the prizes are awarded as follows:

Prize	Number of Winners
$150 cash	1
$30 gift card	4
$3 (money back)	20

(a) **[1 point]** Let Y be the dollar value of the prize. Define (mathematically) an appropriate random variable X to represent a student's **net gain** from buying one ticket.

(b) **[2 points]** Describe each event in X in words. Write the set Ω.

(c) **[3 points]** Write out the probability mass function (PMF) of X for any $X \in \mathbb{R}$.

(d) **[2 points]** What is the probability that that the student will have a positive net gain?

Question 3 [11 points]

(a) **[5 points]** Let $X \sim Binom(12, \theta)$. What value of θ maximizes $\mathbb{P}(X = 11)$?

(b) **[2 points]** A mystery novel book club has 20 members. 8 of them have secretly already read this month's selection, and know the ending. The organizer randomly calls on 6 members during the meeting to share their predictions how the novel ends. What is the probability that no more than 2 of the 6 people know how the story ends?

(c) **[4 points]** A barista makes 12 vanilla lattes during a rush. For each drink, she independently has a 30% chance of forgetting to add the vanilla syrup. The shift manager will only intervene if 3 or more drinks are missing the syrup — otherwise he assumes it's just customer preference. A regular customer arrives and orders one of the 12 drinks at random after they've all been made. What is the probability that the manager does not intervene, but the regular customer's drink is still missing the syrup?