STAT 302: Assignment 3

Summer 2026. Due June 10, 11:59pm on Gradescope.

Question 1 [9 points]

In any given 15 minute block of time, the R4 shows up with the "SORRY, BUS FULL" sign 5 times. Assume the arrival times of full buses are independent. Let X be the time in minutes between full buses.

(a) [**2 points**] Write out the CDF of X. Be sure to include the support.

(b) [**3 points**] What is the variance of the time between full buses? Show the full calculation.

(c) [**4 points**] What is the probability that the total time for 2 consecutive full buses to arrive is less than 12 minutes?

Question 2 [15 points]

The provincial avalanche centre monitors two adjacent weather stations in the Cascade Mountains. Let X be the snow depth (cm) at the lower station and Y be the snow depth (cm) at the upper station on a randomly selected day in January. Their joint PDF is modelled as:

$$f_{X,Y}(x,y) = \frac{1}{3000}(x+2y), \qquad 0 \leq x \leq 10, 0 \leq y \leq 15$$

(a) [**2 points**] On any given day, what is the probability that the snow depth at the lower station is greater than 5cm and the snow depth of the upper station is less than 9cm?

(b) [**2 points**] Calculate the marginal pdf $f_X(x)$.

(c) [**2 points**] Derive the joint CDF for $0 \leq x \leq 10, 0 \leq y \leq 15$

(d) [**2 points**] On any given day, what is the probability that the snow depth at the lower station is more than 3cm greater than snow depth at the upper station?

(e) [**3 points**] What is the conditional distribution $f_{X|Y(x|y)}$?

(f) [**4 points**] Are X and Y independent? Discuss why or why not, providing mathematical reasoning.

Question 3 [11 points]

The joint PMF of discrete random variables X and Y is given by:

$X\backslash Y$	$Y=0$	$Y=1$	$P(X=x)$
$X=0$	0.30	0.05	0.35
$X=1$	0.20	0.25	0.45
$X=2$	0.10	0.10	0.20
$P(Y=y)$	0.60	0.40	1.00

(a) [**2 points**] Compute the conditional distribution $\mathbb{P}_{Y|X}(y \mid X=1)$ for all $y \in \mathbb{R}$.

(b) [**3 points**] Calculate $\mathbb{P}(Y=1|X \geq 1)$. Show your work.

(c) [**3 points**] Compute the conditional distribution $P_{X|Y}(x \mid Y=0)$ for all $x \in \mathbb{R}$.

(d) [**2 points**] Are X and Y independent? Justify your answer mathematically by checking at least two conditions from the definition of independence.

(e) [**3 point**] Compute $\mathbb{E}(XY)$.