STAT 302: Assignment 4

Summer 2026. Due June 17, 11:59pm on Gradescope.

Question 1 [14 points]

Consider the following joint distribution between random variables X and Y.

(X , Y)	1	2	3
0	0.10	0.15	0.05
1	0.20	0.30	0.10
2	0.05	0.03	0.02

(and of course, $\mathbb{P}(X, Y) = 0$ otherwise.)

(a) **[3 points]** What is $\mathbb{E}[X \mid Y]$?

(b) **[4 points]** Calculate the correlation between X and Y.

(c) **[3 points]** Let $Z = X + Y$. Find the MGF of Z. Simplify your answer as much as possible.

(d) **[4 points]** Use your MGF from part (c) to find the variance of Z. *Hint: start by finding* $\mathbb{E}(Z)$ *and* $\mathbb{E}(Z^2)$.

Question 2 [8 points]

Let X and Y be continuous random variables with joint pdf $f_{X,Y}(x, y) = 3x I_{[0,1]}(x) I_{[0,x]}(y)$.

(a) **[4 points]** Compute the covariance between X and Y.

(b) **[4 points]** Find the marginal MGF of X, defined as $m_X(t) = \mathbb{E}[e^{tx}]$. Show all of your work.

Question 3 [13 points]

Let $\Theta \sim \text{Unif}(0, 1)$ and let $X \mid \Theta \sim \text{Binom}(4, \Theta)$.

(a) **[3 points]** What is $\mathbb{E}[X]$? You may refer to the general result of the expected values of any distribution that is found at the end of this assignment.

(b) **[4 points]** What is $\text{Var}[X]$? You may refer to the general result of the expected values of any distribution that is found at the end of this assignment.

(c) **[3 points]** Let $g(\Theta) = \mathbb{E}[X \mid \Theta]$ and consider the transformed variable $Y = e^{g(\Theta)}$. Establish a lower bound on $\mathbb{E}[Y]$. Show all of your work and the inequality you use. *Hint: $h(x) = e^x$ is a convex function.*

(d) **[3 marks]** Find an upper bound on $P(|X - 2| \geq 3)$. Show your work and name the inequality you use.