

# STAT302 Calculus Practice

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This note has a few questions to help you practice mathematical concepts from calculus that may appear on STAT302 exams.

Q. Evaluate  $\lim_{n \rightarrow \infty} \exp(3 + 4/n)$ .

Q. Evaluate  $\lim_{x \rightarrow 0} \int_0^y \exp(3xt) dt$ .

Q. Evaluate  $\lim_{x \rightarrow 0^+} \log(\exp(3x - y) + \exp(\log \Phi(\frac{1}{x^2})))$  (recall  $\Phi$  is the standard normal CDF)

Q. Evaluate  $\lim_{x \rightarrow \infty} \log\left(\frac{1}{1+e^{-x}}\right)$

Q. Evaluate:  $\lim_{x \rightarrow y} \int_0^\infty x^3 \exp(-x^2 y) dy$ . Be careful here, this one is tricky on purpose.

Q. Evaluate  $\lim_{n \rightarrow \infty} \int_{-n}^n \exp(-(x-5)^2/2) dx$ .

Q. Evaluate  $\frac{d}{dx} \sum_{x=1}^5 f(x)g(5, x)$ . Be careful, this is again tricky on purpose.

Q. Evaluate  $\frac{d}{dx} \sum_{y=1}^5 f(y)g(x, y)$ .

Q. Evaluate  $\frac{d}{dx} \log(1 + e^{-3x})$ .

Q. Show that  $f(x) = x^2$  is convex.

Q. Is  $f(x) = \int_0^1 \exp(sx) ds$  convex?

Q. Is  $f(x) = x^3$  concave, convex, or neither?

Q. Show that  $f(x) = \lim_{n \rightarrow \infty} 3(1 + x/n)^n$  is convex.

Q. Show that the CDF of  $\text{Exp}(\lambda)$  is concave (negative convex).

Q. Let  $f(x) = x^2 e^{-x}$  for  $x \geq 0$ . Find  $f'(x)$  and determine the value(s) of  $x$  where  $f'(x) = 0$ .

Q. Find each of the following derivatives and simplify:  $\frac{d}{dx} [\log(1 + x^2)]$ ,  $\frac{d}{dx} [e^{-x^2/2}]$ .

Q. Let  $f(x, y) = e^{-(x+y)}$  for  $x, y \geq 0$ . Compute  $\frac{\partial f}{\partial x}$  and  $\frac{\partial f}{\partial y}$ . Then verify that  $\frac{\partial^2 f}{\partial x \partial y} = f(x, y)$ .

Q. Find  $\frac{d}{dx} \frac{x}{(1+x)^2}$  and simplify your answer fully.

Q. For  $f(x, y) = 6x$  on the region  $\mathcal{B} = \{(x, y) : 0 \leq x \leq y \leq 1\}$  compute the double integral  $\int \int_{\mathcal{B}} f(x, y) dy dx$ .

Q. Compute  $\int_0^\infty x^2 e^{-x} dx$  using integration by parts.

Q. Given the joint density  $f(x, y) = e^{-y}$  for  $0 \leq x \leq y < \infty$ , find the marginal density of  $X$  and identify its distribution.

Q. Let  $f(x) = x(1-x)^2$  for  $x \in [0, 1]$ . Use calculus to find the mode, i.e., the value of  $x$  that maximizes  $f(x)$ .

Q. Let  $f(x) = x^2$ . Is  $f$  monotone increasing, decreasing, or neither?

Q. Let  $f(x) = x^3$ . Is  $f$  monotone increasing, decreasing, or neither?

Q. Let  $f(x) = \frac{e^{-x}}{1+e^{-x}}$ . Is  $f$  monotone increasing, decreasing, or neither?

Q. Let  $h(s) = \int_0^1 \exp(sx) dx$ . Is  $h$  monotone increasing, decreasing, or neither?

Q. Let  $h(s) = \int_0^{s^2} (-\exp(sx)) dx$ . On the set  $s \geq 0$ , is  $h$  monotone increasing, decreasing, or neither?