Lecture 01

Course Introduction and Set Theory

Grace Tompkins

Last modified — 09 Sep 2026

1 Welcome!

Welcome to STAT 302

Topics (Broadly):

- Set theory
- Probability Rules
- Random Variables and Distributions
- Joint, Marginal, and Conditional Distributions
- Expectation, Variance, Moment Generating Functions
- Inequalities and Convergences

Be prepared to do lots of math (proofs, limits, derivatives, integration, maximization, minimization, transformations).

Welcome to STAT 302

- I am your instructor, Dr. Tompkins
  - Pronouns: she/her/hers
- Education: BSc Math (StFX), MMATH + PhD in Biostatistics (UWaterloo)
- Hobbies: crafting, cozy gaming, travel

Syllabus

- In-class checkpoints (7): 10%
- Midterms (2) : 20% each
- Final Exam: 50%

OR

- In-class checkpoints (7): 10%
- Midterms (2) : 12.5% each
- Final Exam: 65%

(whichever gives you the highest mark!)

Syllabus

In-class Checkpoints (10%)

- There are 7 In-class checkpoints throughout the term.
  - Based on weekly problems assigned for homework
  - ~15 minutes each to solve one short problem in class
- Each checkpoint is worth 10 points, and there are 70 points available to gain. Your score will be calculated as `min(CP1 + CP2 + CP3 + CP4 + CP5 + CP6 + CP7, 50)`.
  - Yes, this means that you can get a 0 on two checkpoints and still get full marks
  - Yes, this means you could get 8/10 on all 7 checkpoints and still get full marks.
  - Yes, this means that you can flop a few checkpoints and not have it greatly impact your grade.
- Can bring a cheat sheet (8.5 x 11 inches or smaller, HANDWRITTEN - no photocopies, no prints, not typed or written on tablets)
- Can bring a non-graphing calculator

Syllabus

In-class Checkpoints

- Friday Sept 25
- Friday October 02
- Friday October 09
- Friday October 30
- Friday November 06
- Friday November 27
- Friday December 04
- **NO CONCESSIONS**. The grading scheme allows you to completely miss two checkpoints and still get full marks, though this is NOT recommended. Plan to write all checkpoints - save the “drops” for if you get sick.
- Emails inquiring about dropping/reweighting checkpoints will be politely pointed to the syllabus.

Syllabus

Exams

- Midterm 1: Friday October 16th
- Midterm 2: Friday November 20
- Final: TBD **do not book travel until exam dates have been announced**
- All closed-book, but you can have a “cheat sheet” (see syllabus for rules)

Syllabus

Exams

- Academic concessions:
  - we do not have enough time for make-up midterms. If you are sick, fill out the academic concession form on Canvas **prior** to the midterm and email it to me. The midterm weight will be shifted to the final. No exceptions.

Self-declared concessions can only be used for once throughout the course.

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-  For example, if a student already submitted a self-declared concession form for Midterm 1 and then requests a concession for Midterm 2, the case will be escalated to Science Advising.
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Office hours

Office hours offer you an opportunity to get specific help from the teaching team to clarify concepts and work through practice problems.

When office hours are busy, they may run more like a tutorial where everyone can listen in on the questions being answered.

Teaching Team Member	Times	Location
Grace (Instructor)	Thursdays, 2pm - 3pm	ESB 3174
Nathan	Mondays, 1pm - 2pm	ESB 3174
Isaac	Wednesdays, 1pm - 2pm	ESB 3174
Carlo	Wednesdays, 11am - 12pm	ESB 1041

Other Course Details

- All content will be posted on this website
- Piazza is also enabled for **peer discussion** - this is **not** a place to ask TAs/me for help with problems. Help each other out!
  - We will monitor and remove inappropriate posts.
- To get help from me/the TAs, come to office hours 😁
  - I will not answer course content questions over email - please only email me for private questions such as accommodations or academic concessions.

Course Policy

- This classroom has a **zero tolerance policy for disrespectful behavior**.
- The content in this course can be challenging - help each other out! Be respectful and kind to your fellow students, TAs, and teaching team.
  - This includes online (Reddit 🔎).
- No recording/taking photos during lecture, please. Everything will be posted online for you.

Questions for me?

Advice

This is more of a **math** course than a **data science** course. It uses calculus, proofs, and other mathematical skills that we expect at a 300-level statistics course.

The biggest barriers to success are leaving exam preparation to the last minute and doing insufficient practice.

- Last term, there was a clear difference between those who studied early and prepared carefully, and those who skipped class and tried to cram on the last day.
- The difficulty level increases quickly and builds, ~~especially in this condensed~~ term. Stay on top of your work and attend office hours if you're struggling early on. ~~This course is also offered in the normal 12-week pace in the Fall.~~

Assumed Knowledge

We will be using many examples involving dice and cards. Unless otherwise stated, you can assume the following:

A standard die (plural is *dice*) 🎲:

- Has 6 sides that are numbered from 1 to 6
- Each side has an equal chance of being rolled (unless it is “loaded”)

Assumed Knowledge

A standard deck of cards 🃏:

- Has 52 cards of four suits:
  - red heart ♥️
  - red diamond ♦️
  - black spade ♠️
  - black club ♣️
- Each suit has 13 cards: Ace (1), 2, 3, 4,…, 9, 10, Jack (11), Queen (12), King (13) (we'll ignore the jokers).
- Half of the cards are red, and half are black.
- When shuffled, each card has an equal chance of being pulled.

Assumed Knowledge

- Calculus and all of your other pre-requisites!
- See Calculus Prep to get an idea of the expectations of this course.

Learning Outcomes: Week 1

After this lecture, students are anticipated to be able to:

- Define and use set operations

2 Set Theory Basics

Sets

- Sets are an (unordered) collection of elements denoted with a capital letter and written with curly brackets

  $G = \{1, 2, 3, 4, 5, 6\}$ is a set

- The $\in$ symbol denotes membership. Elements are typically denoted with lowercase letters.

  $g = 4$ $\quad$ $h = 17$

  $g \in G$ $\quad$ $h \notin G$ (h is not in set G)

- The entire space of possible elements (we will later call this our sample space) is denoted as Ω.

Set Operations

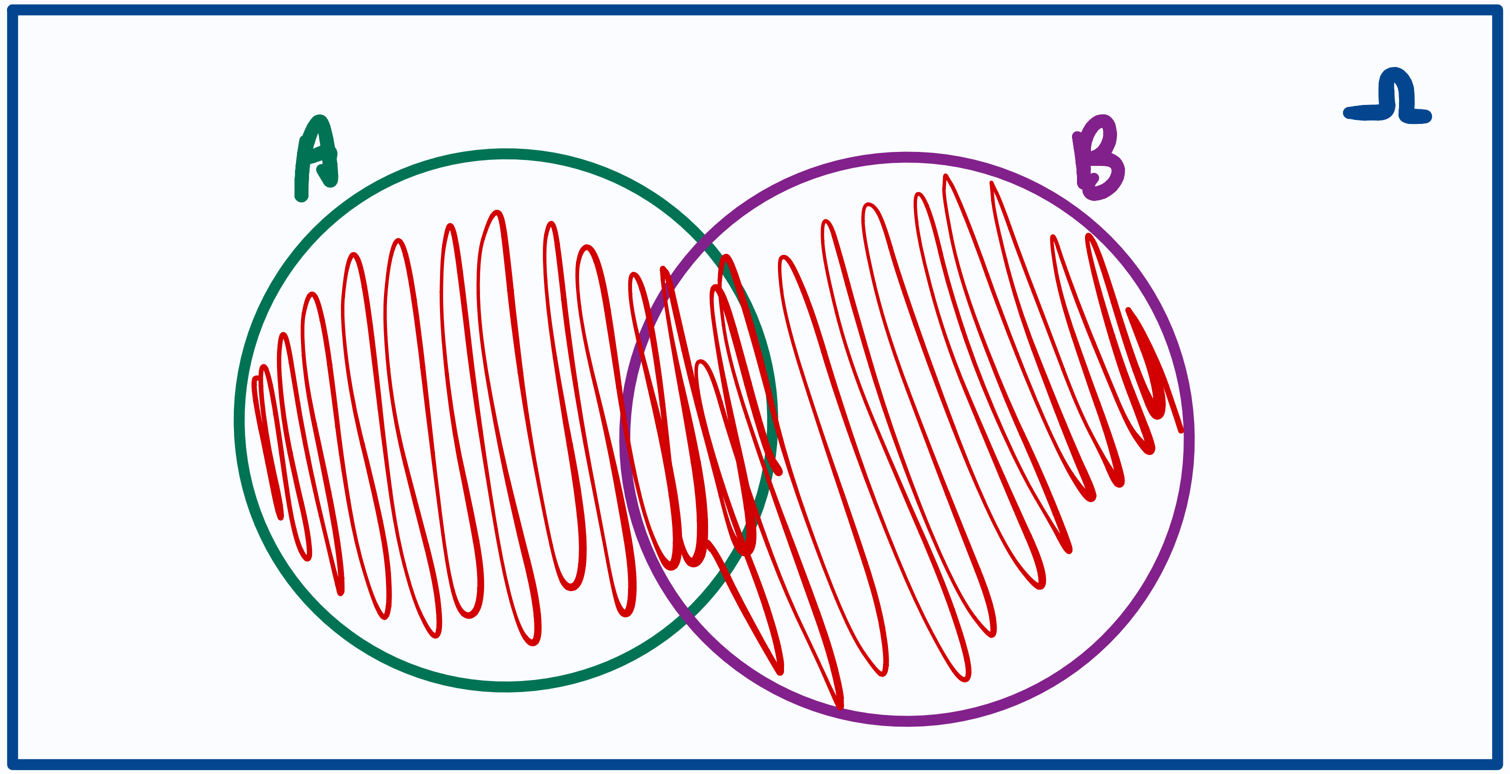
Suppose A, B are events (subsets of Ω).

📖 DEFINITION

Union: $A \cup B$

"union"

$$\omega \in A \cup B \Leftrightarrow \omega \in A \text{ or } \omega \in B$$



$A = \{1, 2, 3\}$

$B = \{2, 3, 5, 9\}$

$A \cup B = \{1, 2, 3, 5, 9\}$

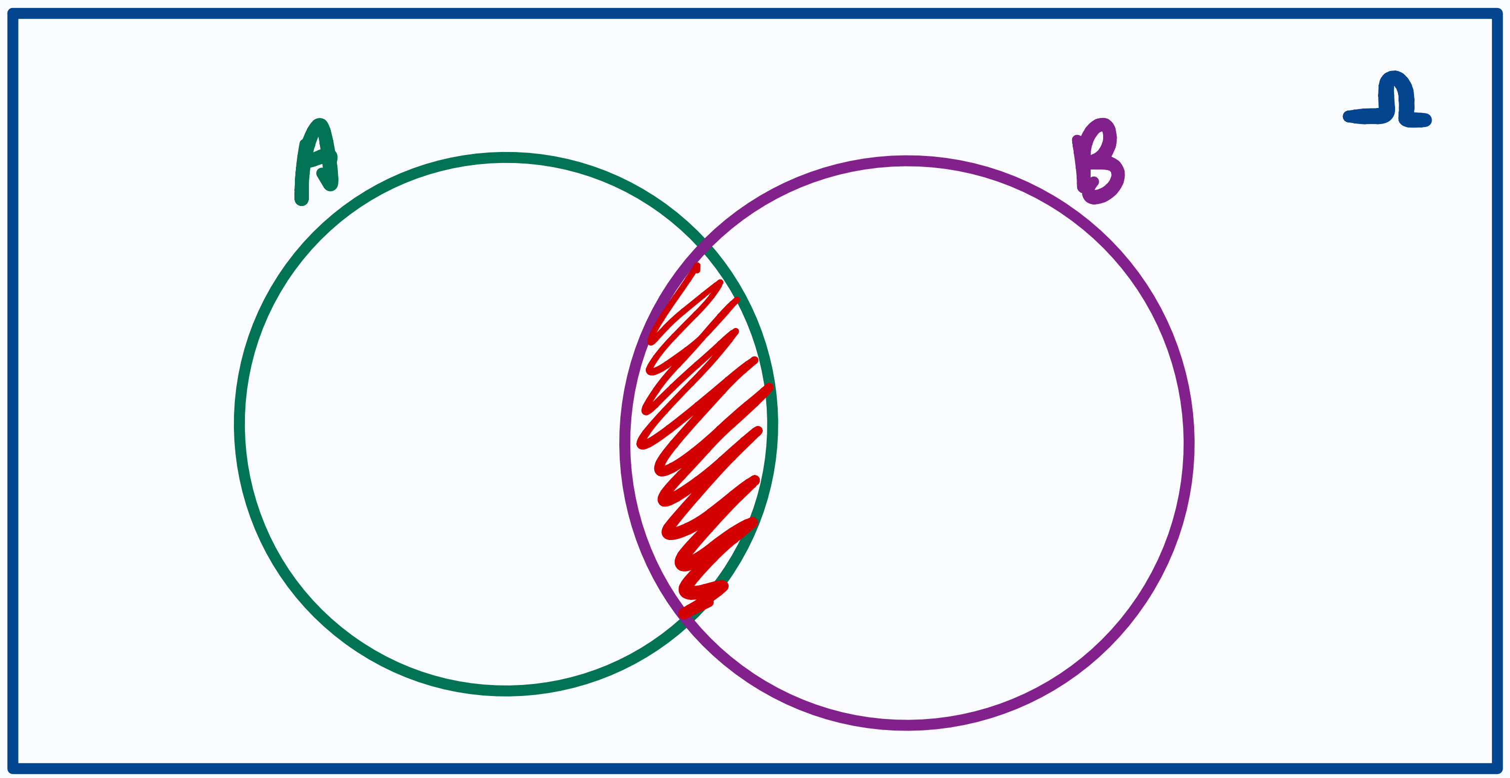
Set Operations

📖 DEFINITION

Intersection: $A \cap B$

"intersect"

$$\omega \in A \cap B \Leftrightarrow \omega \in A \text{ and } \omega \in B$$



$A = \{1, 2, 3\}$

$B = \{2, 3, 5, 9\}$

$A \cap B = \{2, 3\}$

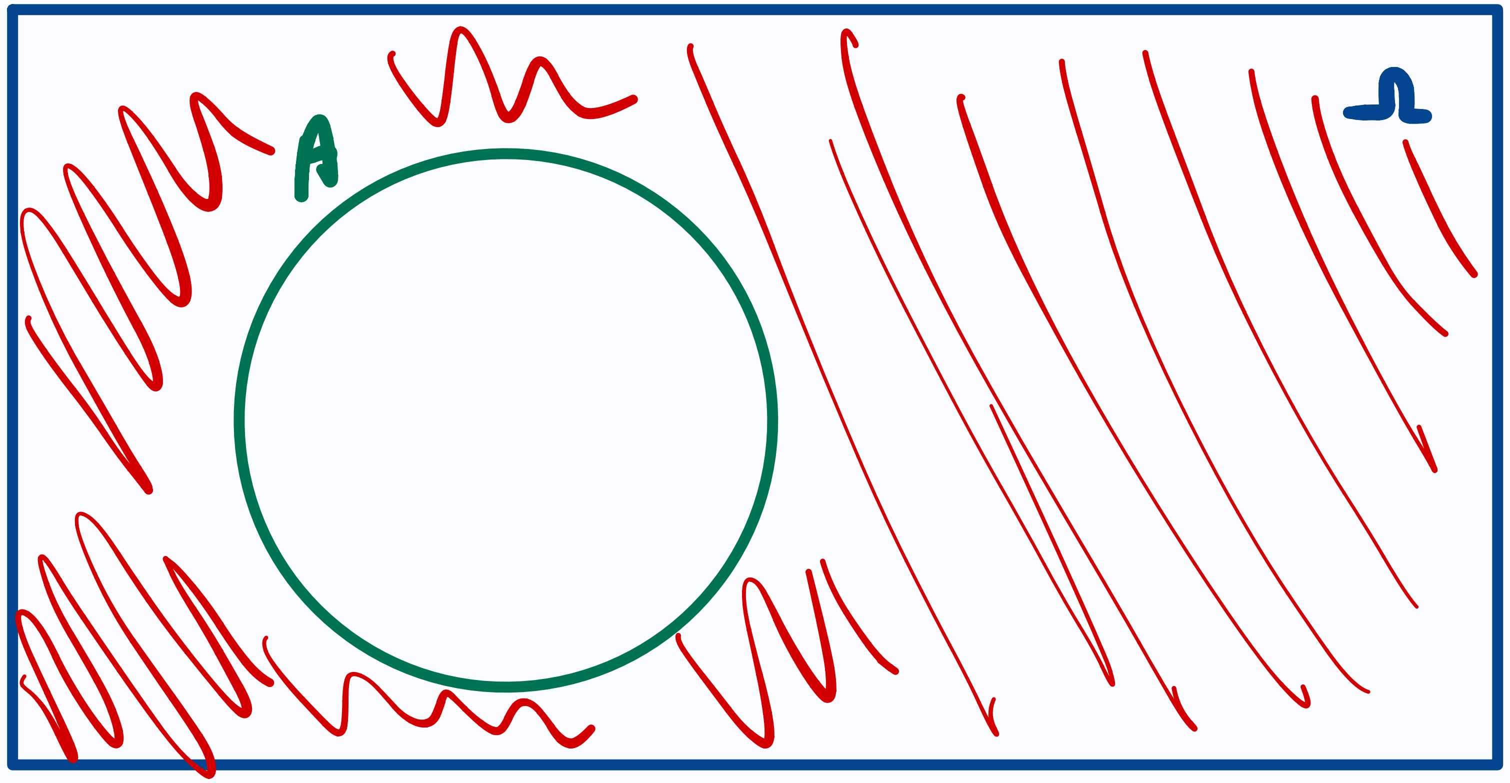
Set Operations

📖 DEFINITION

Complement: A^c

$$\omega \in A^c \Leftrightarrow \omega \notin A$$

↳ opposites (ish)



$\Omega = \mathbb{Z}$ (integers)

$A = \{\text{even integers}\}$

$A^c = \{\text{odd integers}\}$

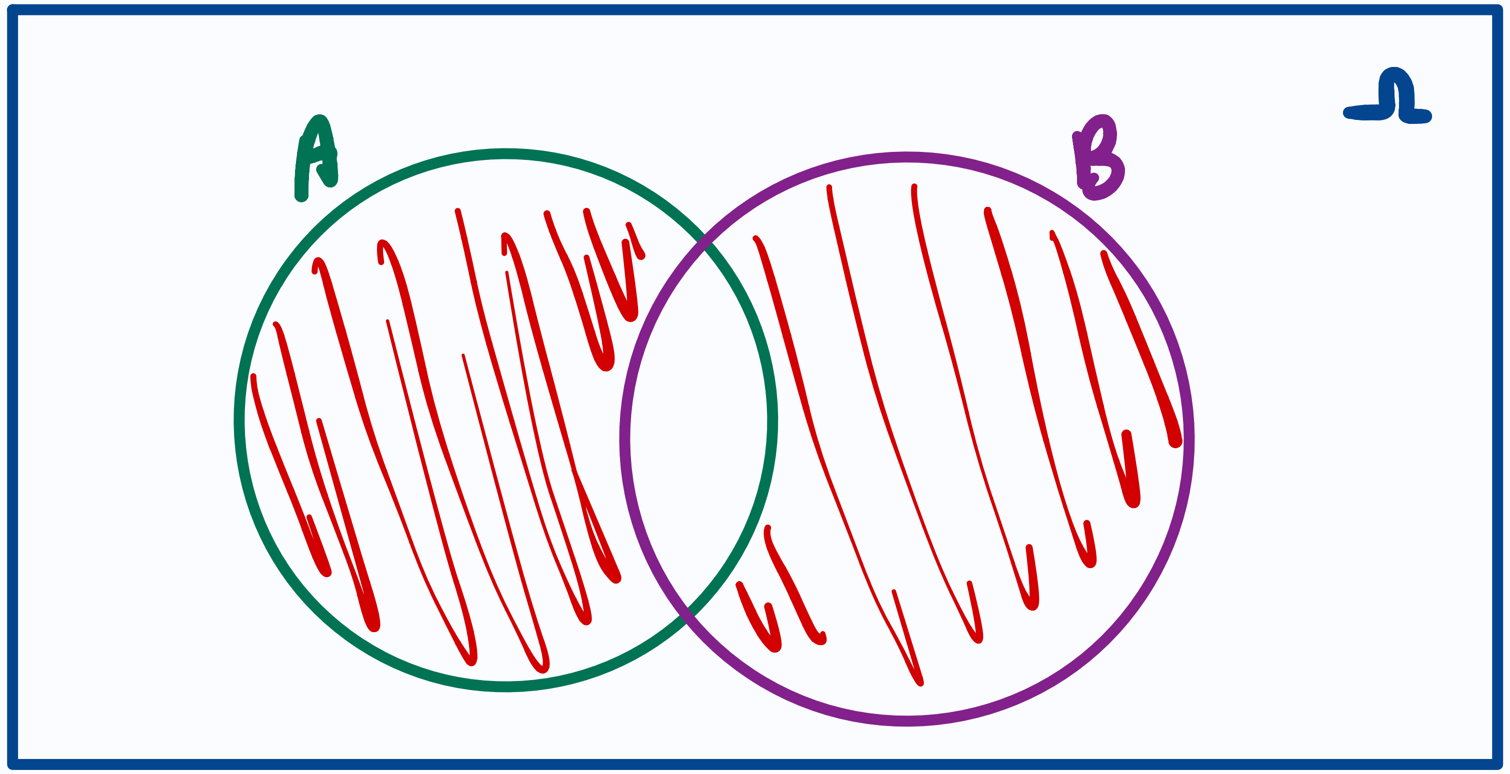
Set Operations

📖 DEFINITION

Symmetric difference: $A \triangle B$

$$A \triangle B = (A \cap B^c) \cup (A^c \cap B)$$

↳ "XOR" in logic



$A = \{1, 2, 3\}$

$B = \{2, 3, 5, 9\}$

$A \triangle B = \{1, 5, 9\}$

Set Operations

📖 DEFINITION

Subset: $A \subseteq B$ is read "A is a subset of B".

This means every element in A appears in B.

$$\forall a, a \in A \implies a \in B$$

A = { grace, david, auden }

B = { robert, david, sheila, grace, audn, mei }

$A \subseteq B$ ✓

Properties of set operations

- Equality
  - $A = B \quad \Leftrightarrow \quad A \subseteq B$ and $B \subseteq A$
- Commutative:
  - $A \cup B \; = \; B \cup A$
  - $A \cap B \; = \; B \cap A$
- Associative:
  - $A \cup B \cup C \; = (A \cup B) \cup C = A \cup (B \cup C)$
  - $A \cap B \cap C \; = \; (A \cap B) \cap C = A \cap (B \cap C)$
- Distributive:
  - $(A \cup B) \cap C \; = \; (A \cap C) \cup (B \cap C)$
  - $(A \cap B) \cup C \; = \; (A \cup C) \cap (B \cup C)$

Laws of Partitioning

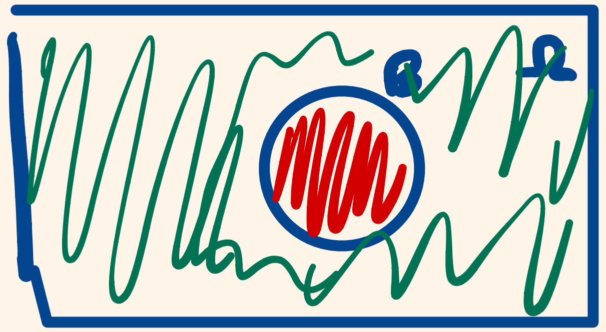
✏️ **EXERCISE: PARTITIONING**

a. Show that $A = (A \cap B) \cup (A \cap B^c)$

Hint: use the fact that $B \cup B^c = \Omega$, $A = A \cap \Omega$

b. Show that $A \cup B = A \cup (B \cap A^c)$

Hint: use the first rule above above to express B in terms of $B \cap A$ and $B \cap A^c$



a) $A = A \cap \Omega$

$= A \cap (B \cup B^c)$

$= (A \cap B) \cup (A \cap B^c)$ by distributive law

Laws of Partitioning

b) $A \cup B = A \cup (B \cap A^c)$

From a) $B = (B \cap A) \cup (B \cap A^c)$

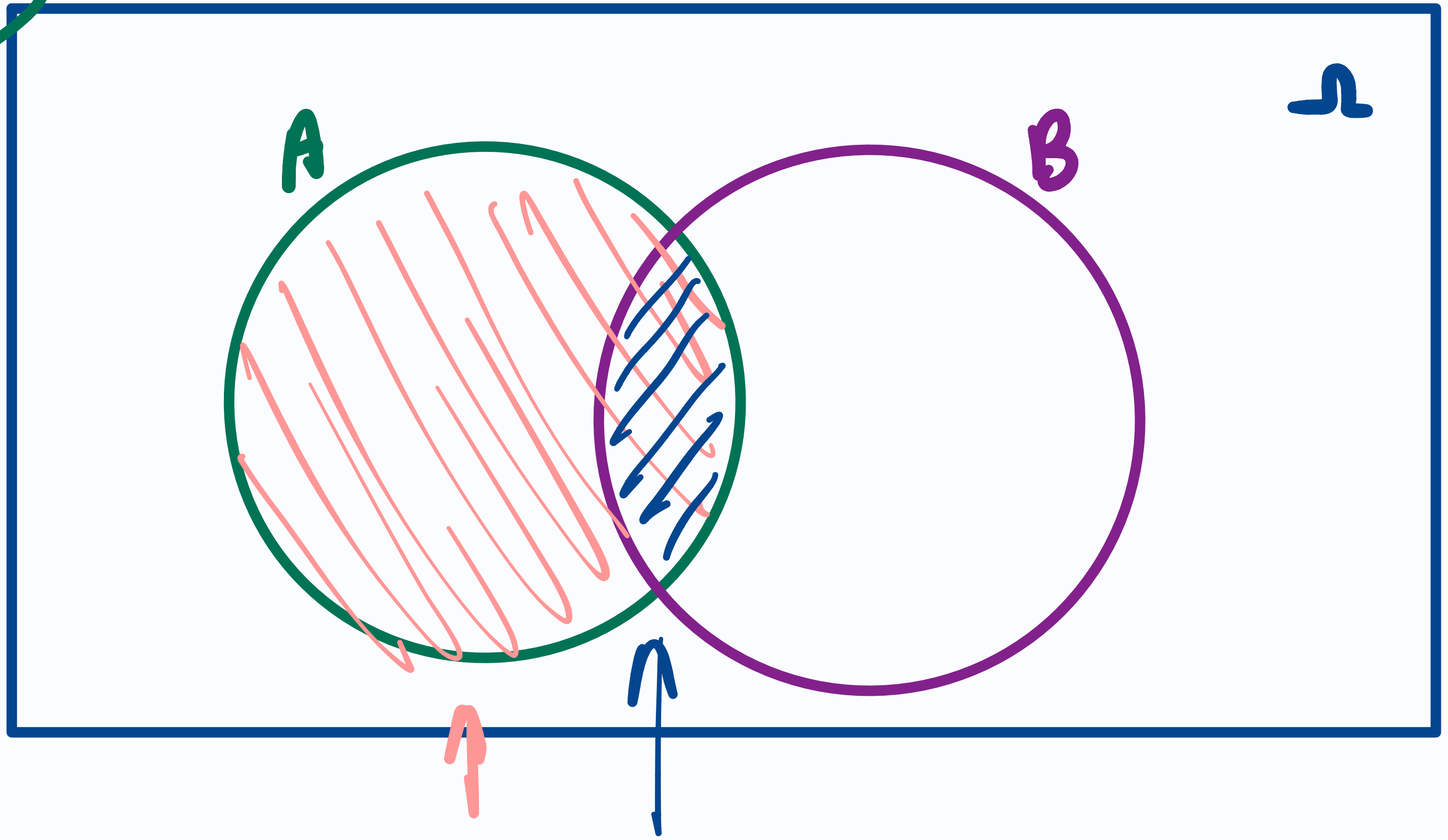
Therefore,

$$A \cup B = A \cup \big((B \cap A) \cup (B \cap A^c)\big)$$

$$= \underline{A \cup (B \cap A)} \cup (B \cap A^c)$$

$$= A \cup (B \cap A^c)$$

Aside:



$A \cup B \cap A = A$

$B \cap A \subseteq A$, therefore $(B \cap A) \cup A = A$

De Morgan's Laws

THEOREM

De Morgan's Laws: For any two events (sets) A and B, we have

$$(A \cup B)^c = A^c \cap B^c$$

To prove the theorem it is sufficient to show that

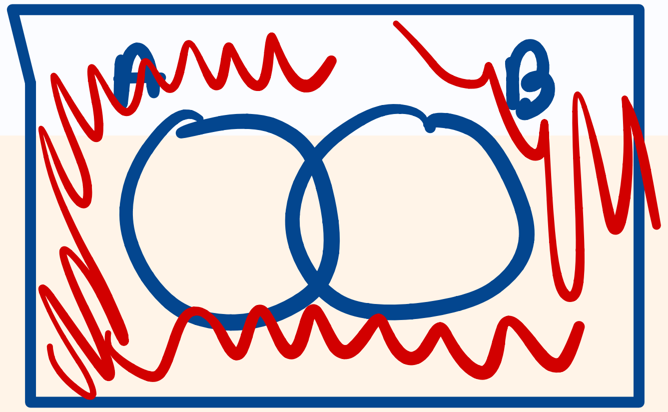
$$① \quad (A \cup B)^c \subseteq A^c \cap B^c$$

and that

$$② \quad A^c \cap B^c \subseteq (A \cup B)^c$$

↳ try ② at home!

Proof of De Morgan's Laws



✏️ **EXERCISE: PROOF OF DE MORGAN'S LAWS**

Prove De Morgan's Laws

① $(A \cup B)^c \subseteq A^c \cap B^c$

Consider any $\omega \in (A \cup B)^c$

$\Rightarrow \omega \notin A$ and $\omega \notin B$

$\Rightarrow \omega \in A^c$ and $\omega \in B^c$

$\Rightarrow \omega \in A^c \cap B^c$

This means for any $\omega \in (A \cup B)^c$, this implies $\omega \in A^c \cap B^c$. (def'n of subset!)

$\therefore (A \cup B)^c \subseteq A^c \cap B^c$

Proof of De Morgan’s Laws

Power Set, Empty Set, Cardinality

The power set of Ω (denoted 2^Ω) is the set of all possible subsets of Ω.

For example, if $\Omega = \{1, 2, 3\}$ then:

$$2^\Omega = \Big\{\varnothing, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\Big\}$$

$|2^\Omega| = 8$

- $\varnothing$ denotes the empty set: $\varnothing = \{\}$.
- $|\cdot|$ denotes the size of a set (number of elements).

*order does not matter!

$\Omega = \{\text{grace}, \text{david}\}$

$2^\Omega = \{\varnothing, \{\text{grace}\}, \{\text{david}\}, \{\text{grace}, \text{david}\}\}$ $|2^\Omega| = 4$

In general, if $|\Omega| = n$, $|2^\Omega| = 2^n$

Size of the Power Set

✏️ **EXERCISE: SIZE OF POWERSET**

If Ω has n elements, what is $|2^{\Omega}|$?

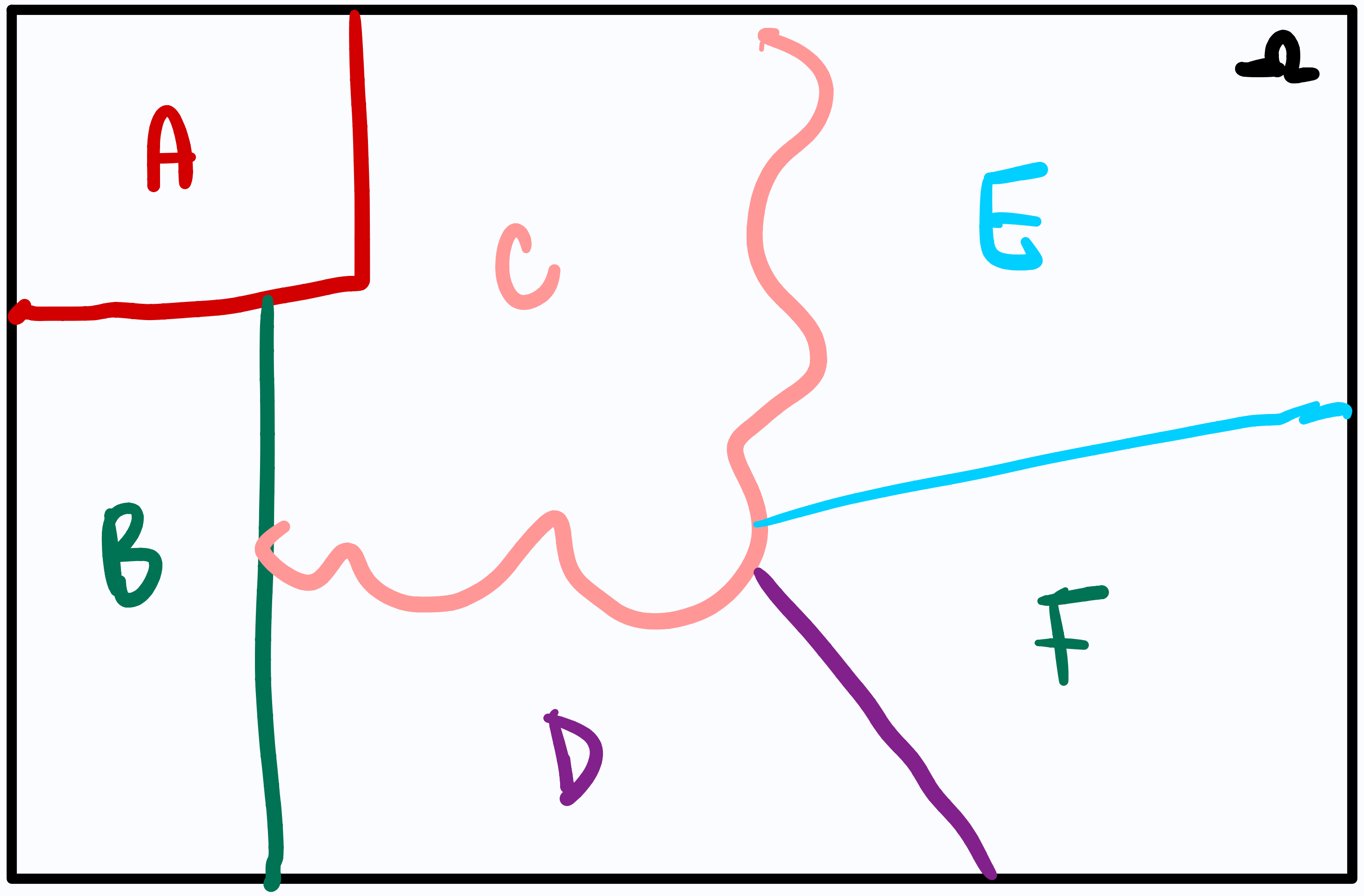
$$|2^{\Omega}| = 2^{|\Omega|} = 2^{n}$$

Size of the Power Set

Partitions

📖 DEFINITION

Partition: A grouping of elements into non-empty, disjoint subsets such that every element in Ω is in exactly one subset.



$\Omega = \{1, 2, 3, 4, 5, 6, 7\}$

Possible partition:

$A = \{1\}$

$B = \{2, 7\}$

$C = \{3, 4, 5, 6\}$

✓