Lecture 1

Course Introduction and Set Theory

Grace Tompkins

Last modified — 12 May 2026

1 Welcome!

Welcome to STAT 302

Topics (Broadly):

- Set theory
- Probability Rules
- Random Variables and Distributions
- Joint, Marginal, and Conditional Distributions
- Expectation, Variance, Moment Generating Functions
- Inequalities and Convergences

Be prepared to do lots of math (proofs, limits, derivatives, integration, maximization, minimization, transformations).

Welcome to STAT 302

- About me:
  - You can call me Grace/Professor/Dr. Tompkins, whatever 🤠
  - Pronouns: she/her/hers
  - I've been at UBC for < 1 year
  - Originally from NS ➡️ moved to Ontario for grad school ➡️ now I'm here!
  - I'm a Biostatistician!
  - Hobbies: crafting, cozy gaming, travel

Syllabus

- Assignments: 4 x 5% each
- Midterm: 30%
- Final Exam: 50%

Syllabus

Assignments (20%)

- 4 assignments, each worth 5%
  - Wed. May 20th, 11:59pm
  - Wed. May 27, 11:59pm
  - Wed. June 10, 11:59pm
  - Wed. June 17, 11:59pm
- Must be **hand written** (tablet or paper) and submitted digitally on Gradescope (linked on Canvas). No typed assignments unless you have accommodations.
  - Write clearly. If the TA cannot read your writing, they will not grade it.
- **Late policy**: 5% deduction per hour late. No submissions after 24 hours (solutions will be posted). With the condensed term, we don't have a lot of wiggle room. Sorry.
- Must submit academic concession form at least 12 hours before due date for extenuating circumstances.

Syllabus

Exams (80%)

- Midterm (Tuesday June 2nd): 30%
- Final: 50%
- All closed-book, but you can have a “cheat sheet”
- Academic concessions:
  - we do not have enough time for make-up midterms. If you are sick, fill out the academic concession form on Canvas **prior** to the midterm and email it to me. The midterm weight will be shifted to the final. No exceptions.

The in-class exercises and weekly assignment problems are designed to prepare you for the kinds of questions we ask on exams (so… actually do them! They’re there to help you practice. Don’t rely on solutions you find online or AI assistance.)

Weekly Expectations

This is a short term - we move fast!

- **Pre-class reading**: 2 - 3 hours. Come to lecture prepared to do problems!
- **Weekly Assignments**: 4 - 5 hours
- **Class meetings**: 6 hours (2 hours x 3 times a week)

Office hours

Office hours offer you an opportunity to get specific help from the teaching team to clarify concepts and work through practice problems.

When office hours are busy, they may run more like a tutorial where everyone can listen in on the questions being answered.

Teaching Team Member	Times	Location
Grace (Instructor)	Tues/Wed, 12:00pm - 1:00pm	(This week only) ESB 1041*
Isaac	Tuesdays, 12:00pm - 1:00pm	(This week only) ESB 1041*
Mohammad	Thursdays, 4:00pm - 5:00pm	ESB 3174
Nathan	Fridays, 3:00pm - 4:00pm	ESB 3174

*ESB 4192 after May 18

Other Course Details

- All content will be posted on this website
- **Canvas is currently down** 💀
  - I have the same amount of information as you, and anticipate it to be back up next week.
  - Course communication will eventually be through Canvas announcements. Ensure your email notifications are on.
  - Until then, and Piazza and email will be used for announcements. Please ensure you've joined using the link I've emailed to you.
  - **LATE ENTRANTS: please email me for access to Piazza and Gradescope!**

Other Course Details

- Piazza is also enabled for **peer discussion** - this is **not** a place to ask TAs/me for help with problems. Help each other out!
  - We will monitor and remove inappropriate posts.
- To get help from me/the TAs, come to office hours 😁
  - I will not answer course content questions over email - please only email me for private questions such as accommodations or academic concessions.

Course Policy

- This classroom has a **zero tolerance policy for disrespectful behavior**.
- The content in this course can be challenging, and the summer term is face-paced. Help each other out! Be respectful and kind to your fellow students, TAs, and teaching team.
  - This includes online (I am on Reddit 🔎).
- No recording/taking photos during lecture, please. Everything will be posted online for you.

Questions for me?

Advice

This is more of a **math** course than a **data science** course. It uses calculus, proofs, and other mathematical skills that we expect at a 300-level statistics course.

The biggest barriers to success are leaving exam preparation to the last minute and doing insufficient practice.

- Last term, there was a clear difference between those who studied early and prepared carefully, and those who skipped class and tried to cram on the last day.
- The difficulty level increases quickly and builds, especially in this condensed term. Stay on top of your work and attend office hours if you're struggling early on. This course is also offered in the normal 12-week pace in the Fall.

Assumed Knowledge

We will be using many examples involving dice and cards. Unless otherwise stated, you can assume the following:

A standard die (plural is *dice*) 🎲:

- Has 6 sides that are numbered from 1 to 6
- Each side has an equal chance of being rolled (unless it is “loaded”)

Assumed Knowledge

A standard deck of cards 🃏:

- Has 52 cards of four suits:
  - red heart ♥️
  - red diamond ♦️
  - black spade ♠️
  - black club ♣️
- Each suit has 13 cards: Ace (1), 2, 3, 4,…, 9, 10, Jack (11), Queen (12), King (13) (we'll ignore the jokers).
- Half of the cards are red, and half are black.
- When shuffled, each card has an equal chance of being pulled.

Assumed Knowledge

- Calculus and all of your other pre-requisites!
- See Calculus Prep to get an idea of the expectations of this course.

Learning Outcomes: Lecture 1

After this lecture, students are anticipated to be able to:

- Define and use set operations

2 Set Theory Basics

Sets

- Sets are an (unordered) collection of elements denoted with a capital letter and written with curly brackets

$G = \{1, 2, 3, 4, 5, 6\}$ is a set

- The $\in$ symbol denotes membership. Elements are typically denoted with lowercase letters.

$h = 4$

$h \in G$

$k = 17$

$k \notin G$

← not an element of

- The entire space of possible elements (we will later call this our sample space) is denoted as Ω.

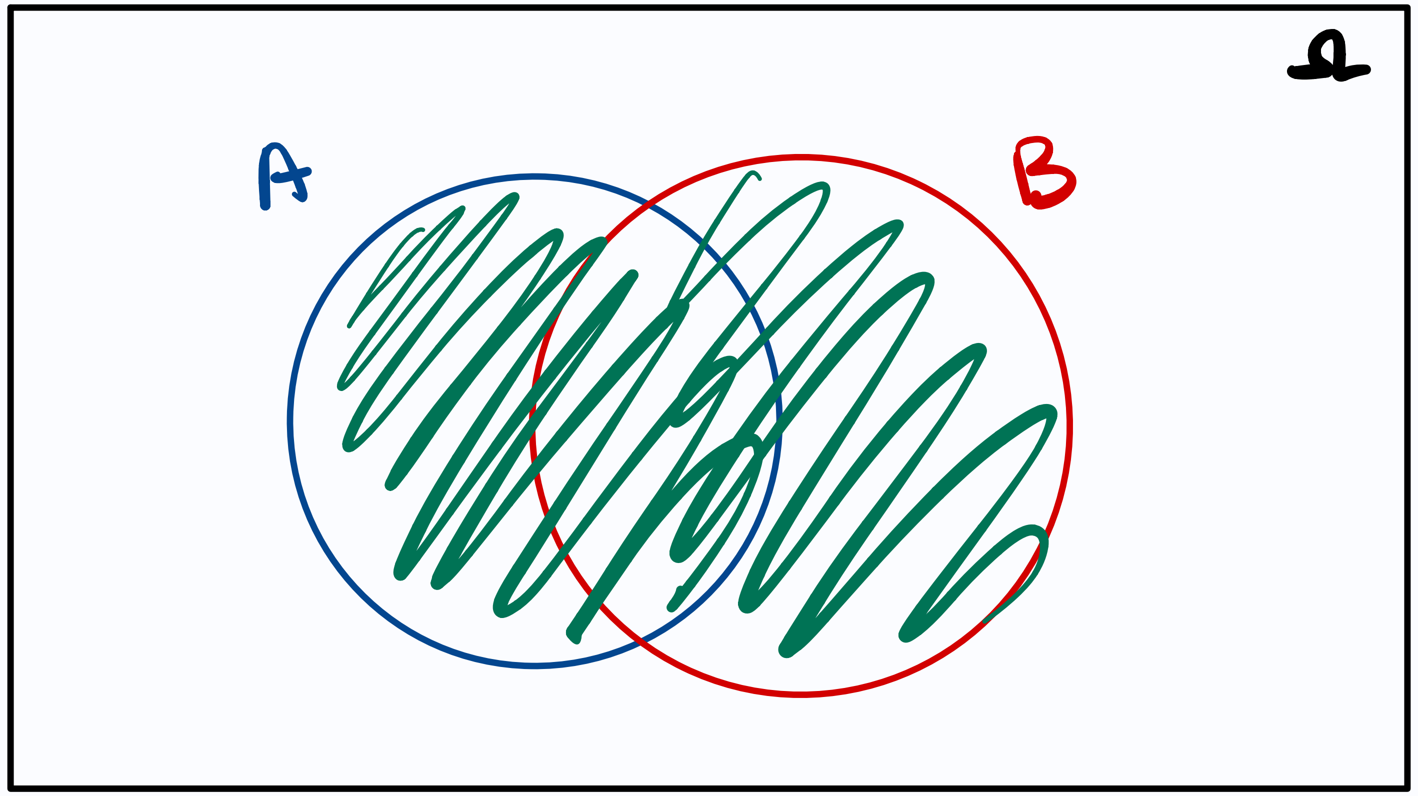
Set Operations

Suppose A, B are events (subsets of Ω).

📖 DEFINITION

Union: $A \cup B$

$$\omega \in A \cup B \Leftrightarrow \omega \in A \text{ or } \omega \in B$$



$A = \{1, 2, 3\}$

$B = \{4, 5, 6\}$

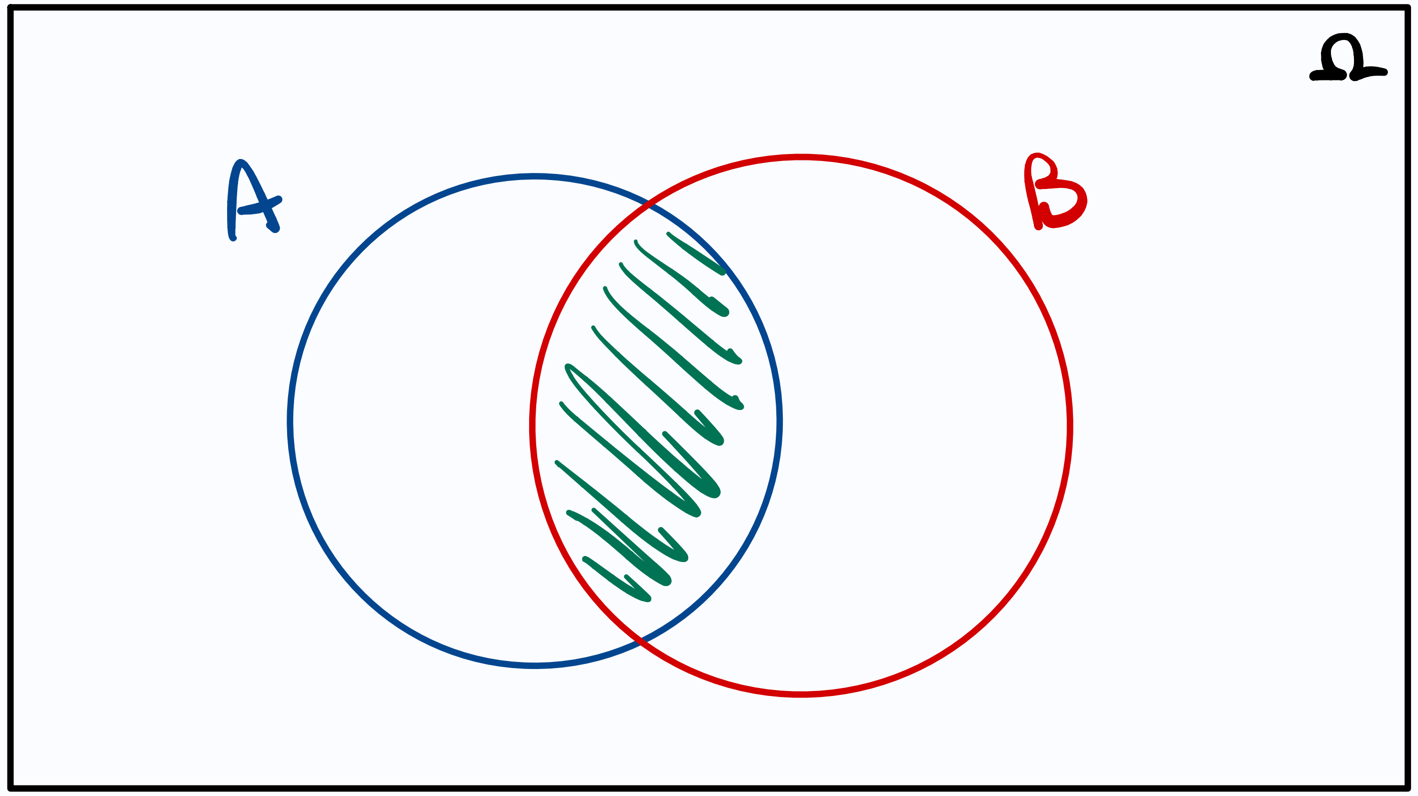
$A \cup B = \{1, 2, 3, 4, 5, 6\}$

Set Operations

📖 DEFINITION

Intersection: $A \cap B$

$$\omega \in A \cap B \Leftrightarrow \omega \in A \text{ and } \omega \in B$$



A = {1, 2, 3}

B = {2, 3, 4}

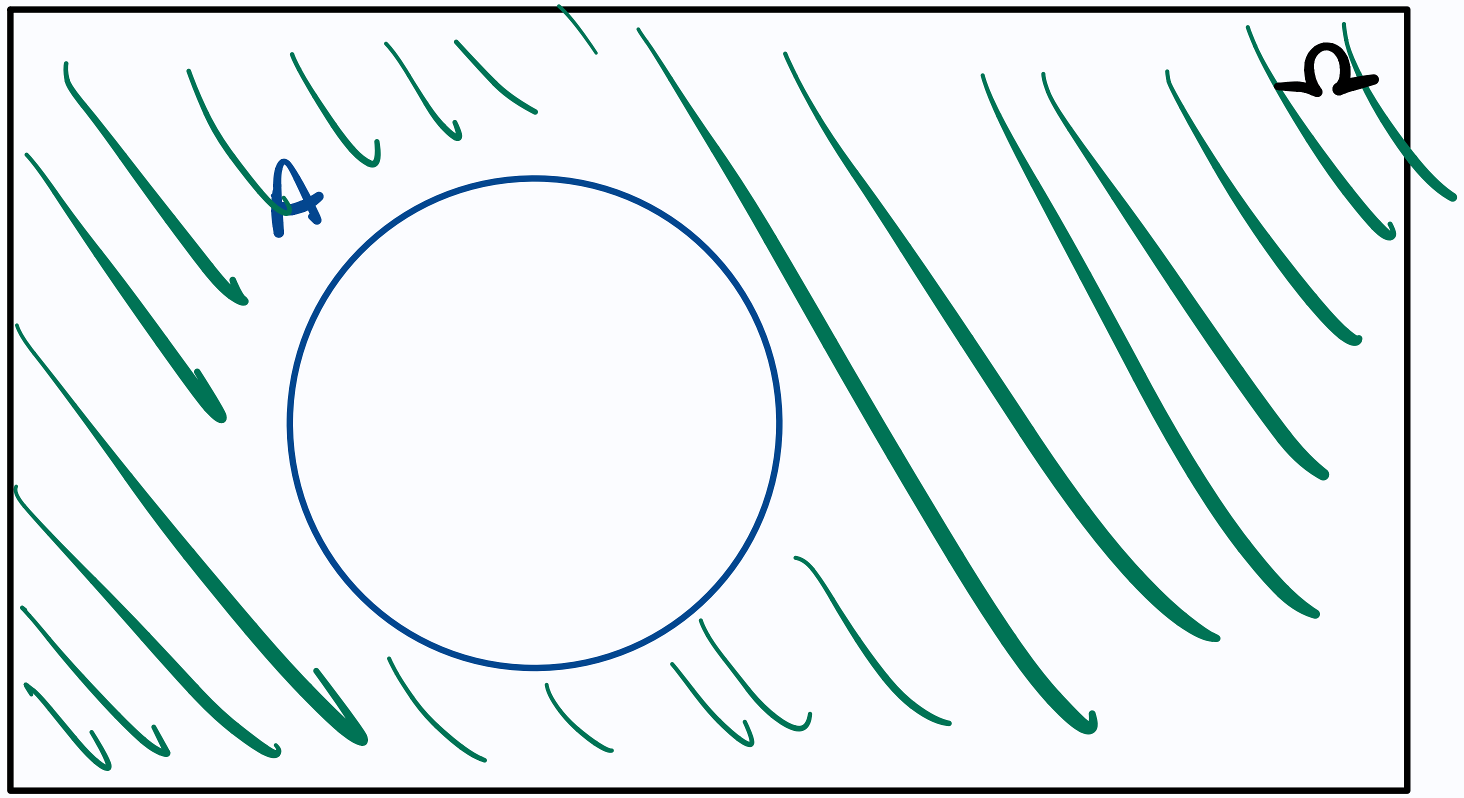
A ∩ B = {2, 3}

Set Operations

📖 DEFINITION

Complement: A^c

$$\omega \in A^c \Leftrightarrow \omega \notin A$$



let $\Omega = \mathbb{Z}$ (integers)

$A = \{\ldots -4, -2, 0, 2, 4, \ldots\}$ (evens)

$A^c = \{\ldots -3, -1, 1, 3, \ldots\}$ (odds)

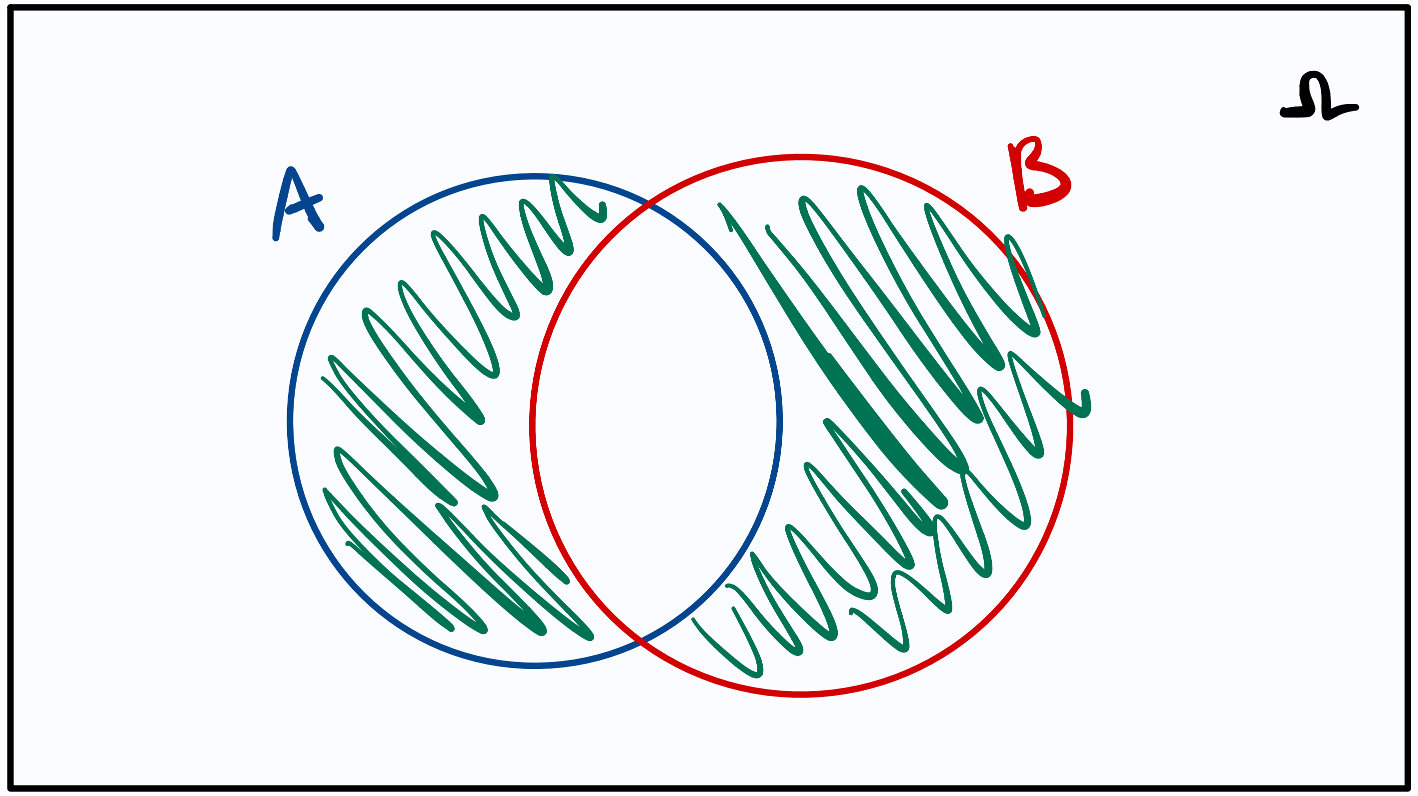
Set Operations

📖 DEFINITION

Symmetric difference: $A \triangle B$

$$A \triangle B = (A \cap B^c) \cup (A^c \cap B)$$

↳ "XOR" in logic!



$A = \{1, 2, 3, 4\}$

$B = \{2, 3, 4, 5, 6\}$

$A \triangle B = \{1, 5, 6\}$

Set Operations

📖 DEFINITION

Subset: $A \subseteq B$ is read "A is a subset of B".

This means every element in A appears in B.

$$\forall a, a \in A \implies a \in B$$

A = { grace, tom, owen }

B = { tom, jackson, grace, sheila, gertrude, owen }

$A \subseteq B$ here

Properties of set operations

- Equality
  - $A = B \quad \Leftrightarrow \quad A \subseteq B \text{ and } B \subseteq A$
- Commutative:
  - $A \cup B \;=\; B \cup A$
  - $A \cap B \;=\; B \cap A$
- Associative:
  - $A \cup B \cup C \;=\; (A \cup B) \cup C = A \cup (B \cup C)$
  - $A \cap B \cap C \;=\; (A \cap B) \cap C = A \cap (B \cap C)$
- Distributive:
  - $(A \cup B) \cap C \;=\; (A \cap C) \cup (B \cap C)$
  - $(A \cap B) \cup C \;=\; (A \cup C) \cap (B \cup C)$

Laws of Partitioning

✏️ **EXERCISE: PARTITIONING**

a. Show that $A = (A \cap B) \cup (A \cap B^c)$

Hint: use the fact that $B \cup B^c = \Omega$

b. Show that $A \cup B = A \cup (B \cap A^c)$

Hint: use the first rule above above to express B *in terms of* $B \cap A$ *and* $B \cap A^c$

a) $A = A \cap \Omega$

$= A \cap (B \cup B^c)$

$= (A \cap B) \cup (A \cap B^c)$ (distributive law)

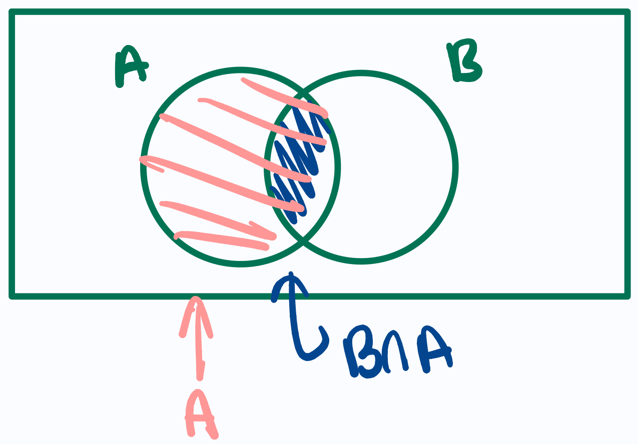
Laws of Partitioning

b) $A \cup B = A \cup [(B \cap A) \cup (B \cap A^c)]$

$= A \cup (B \cap A) \cup (B \cap A^c)$ (assoc. prop)

Then

As $B \cap A \subseteq A$, $A \cup (B \cap A) = A$



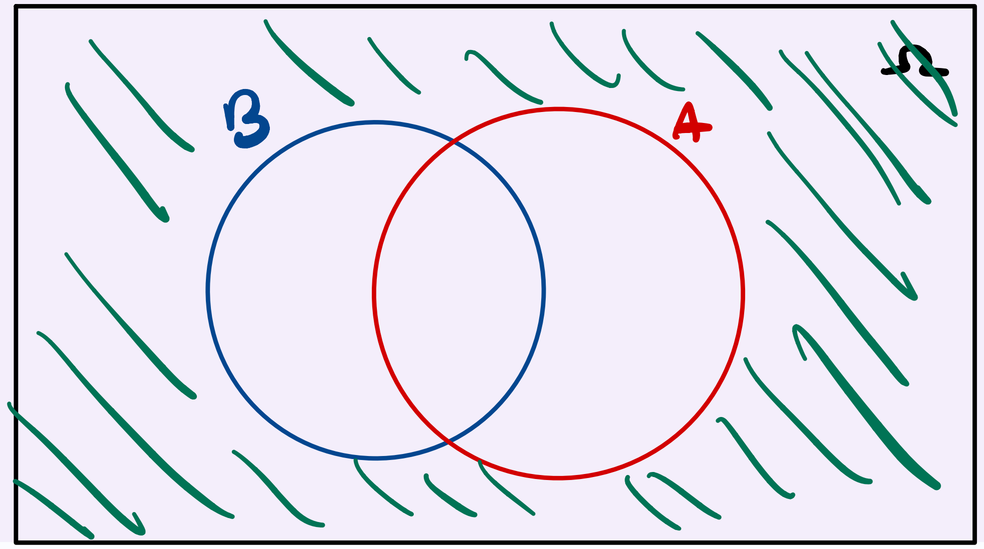
$A \cup B = A \cup (B \cap A^c)$

De Morgan’s Laws

THEOREM

De Morgan’s Laws: For any two events (sets) A and B, we have

$$(A \cup B)^c = A^c \cap B^c$$



To prove the theorem it is sufficient to show that

① $$(A \cup B)^c \subseteq A^c \cap B^c$$

and that

② $$A^c \cap B^c \subseteq (A \cup B)^c$$

Proof of De Morgan's Laws

✏️ **EXERCISE: PROOF OF DE MORGAN'S LAWS**

Prove De Morgan's Laws

① Consider any $\omega \in (A \cup B)^c$

$\Rightarrow \omega \notin A$ and $\omega \notin B$

$\Rightarrow \omega \in A^c$ and $\omega \in B^c$

$\Rightarrow \omega \in A^c \cap B^c$

This means for $\omega \in (A \cup B)^c \Rightarrow \omega \in A^c \cap B^c$

$\therefore (A \cup B)^c \subseteq A^c \cap B^c$

Proof of De Morgan's Laws

Try part ② at home!

Power Set, Empty Set, Cardinality

The power set of Ω (denoted 2^Ω) is the set of all possible subsets of Ω.

For example, if $\Omega = \{1, 2, 3\}$ then:

$$2^\Omega = \Big\{\varnothing, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\Big\}$$

- $\varnothing$ denotes the empty set: $\varnothing = \{\}$.
- $|\cdot|$ denotes the size of a set (number of elements).

$|\Omega| = 3$

$|2^\Omega| = 2^3 = 8$

Size of the Power Set

✏️ **EXERCISE: SIZE OF POWERSET**

If Ω has n elements, what is $|2^\Omega|$?

$2^{|\Omega|}$!

Ex: $\Omega = \{a, b\}$ $\quad |\Omega| = 2$

$2^\Omega = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

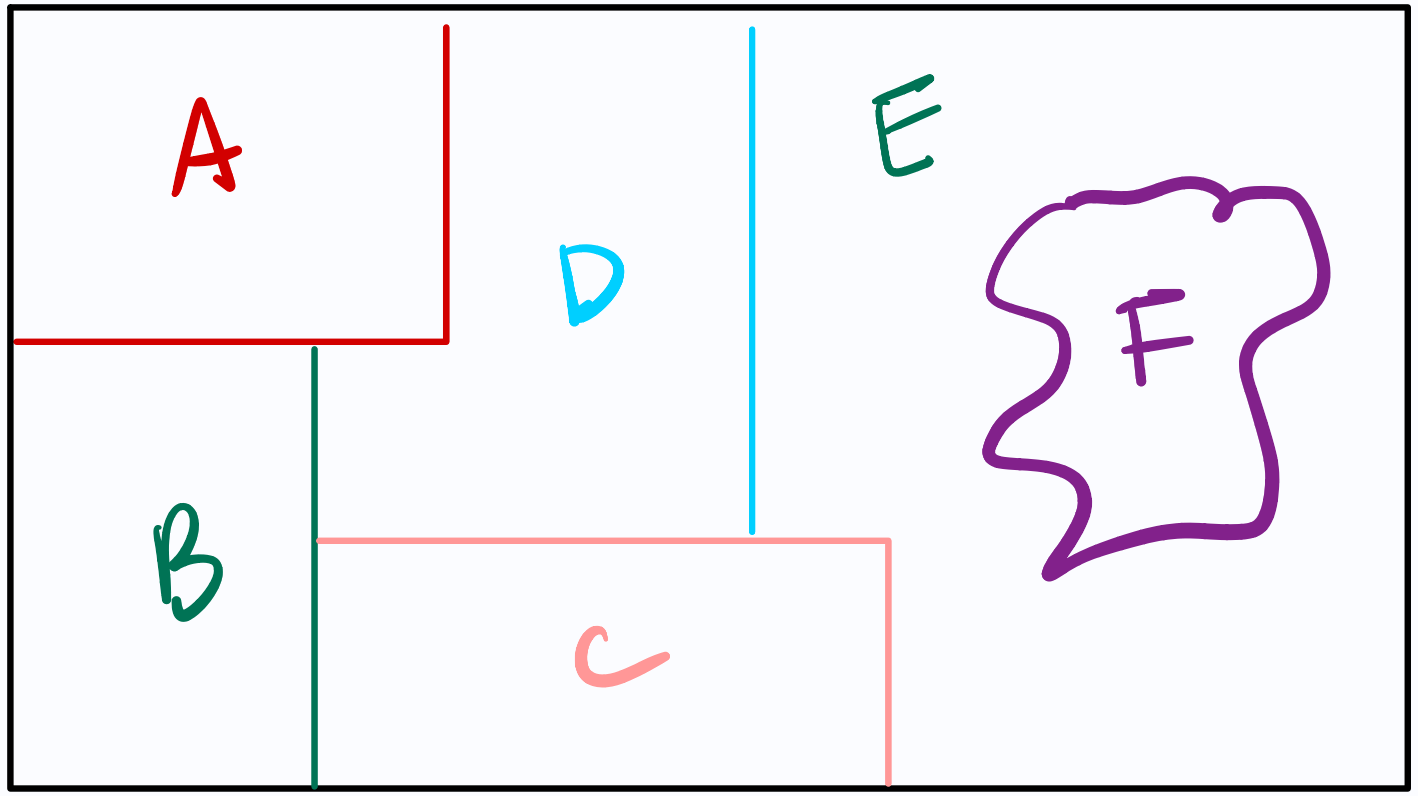
$|2^\Omega| = 2^{|\Omega|} = 2^2 = 4$

Size of the Power Set

Partitions

📖 DEFINITION

Partition: A grouping of elements into non-empty, disjoint subsets such that every element in Ω is in exactly one subset.



$\Omega = \{1, 2, 3, 4, 5, 6, 7, 8\}$

Possible partition:

$A = \{1\}$

$B = \{6, 7, 8\}$

$C = \{3\}$

$D = \{2, 4, 5\}$

To Do:

- Read Chapters 1.1 - 1.3 before next class!
- Keep this week's Assignment due date on your radar (Wednesday May 20, 11:59pm)

Next Class: More probability (formally!)

Take care!