

Lecture 9

Continuous Distributions

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Learning Outcomes

By the end of this lecture, students are anticipated to be able to:

- Define a probability density function
- Use the continuous PDFs to solve for probabilities

1 Continuous Random Variables

Continuous Random Variables

DEFINITION

A RV X is **continuous** if

$$\mathbb{P}(X = x) = 0,$$

for every $x \in \mathbb{R}$.

- This is the most rigorous way to define continuous RVs, but it doesn't provide much intuition.
- We need a bit more to make our intuition match the math.

Absolutely Continuous RVs and Density Functions

DEFINITION

A function $f : \mathbb{R} \rightarrow \mathbb{R}$ is called a **density function** if $f(x) \geq 0$, $\forall x \in \mathbb{R}$ and $\int f(x)dx = 1$.

DEFINITION

A RV X is **absolutely continuous** if there exists a **density function** f such that

$$\mathbb{P}(a \leq X \leq b) = \int_a^b f(x)dx$$

whenever $a \leq b$.

We call such a density function a **probability density function** (PDF) and will often use the notation $f_X(x)$ to denote its relationship to X .

Probability of $a \leq X \leq b$ visually

Absolutely Continuous RVs

THEOREM

Let X be an absolutely continuous random variable. Then X is a continuous random variable (i.e., $\mathbb{P}(X = a) = 0$ for all $a \in \mathbb{R}$)

PROOF

$$\mathbb{P}(X = a) = \mathbb{P}(a \leq X \leq a) = \int_a^a f(x)dx = 0$$

Note

Note: the converse is not necessarily true. That is, not all continuous distributions you will come across in statistics (in general) are **absolutely** continuous. We will stick to absolutely continuous distributions in this course.

Probability of $X = a$ visually

Comparison to Discrete RVs

- Consider some set $A \subset \mathbb{R}$.
- Let X be a random variable.

Probability of A for discrete RV:

$$\mathbb{P}(X \in A) = \sum_{x \in A} p_X(x).$$

Probability of A for an (absolutely) continuous RV:

$$\mathbb{P}(X \in A) = \int_A f_X(x) dx.$$

Continuous PDFs

💡 EXAMPLE

Let X be a RV with PDF

$$f_X(x) = 2x^{-3}I_{[1,\infty)}(x).$$

- a. Is f a valid density function?
- b. What is $\mathbb{P}(X = 2)$?
- c. What is $\mathbb{P}(0 < X < 2)$?
- d. Suppose we defined $g(x) = 2x^{-3}$ (no indicator function). Is $g(x)$ a density function? Does it have the same support?

Continuous PDFs

Continuous PDFs

EXERCISE: EXAMPLE CONTINUOUS RV

Let $f_X(x) = \frac{1}{5} I_{[0,5]}(x)$.

- a. What is $\mathbb{P}(X \leq 3)$?
- b. What is $\mathbb{P}(X < 3)$?
- c. What is $\mathbb{P}(-4 < X < 10)$?
- d. What is $\mathbb{P}(X \geq 2)$?

Continuous PDFs

Reminders

- No class on Monday (thanksgiving)
- Midterm 1 on Friday October 16th (all material up to today's class)
- Start prepping your cheat sheet
 - MUST BE HAND WRITTEN! No photo copies, typed, or tablet-written and printed sheets allowed
 - 8.5 inches x 11 inches or less
 - You can have one single page, and can write on both sides

Checkpoint 3

You may have:

- HANDWRITTEN Cheat Sheet
- Non-graphing calculator
- Writing materials

Phones and laptops put away, please! You can have 15 minutes.