Statistics practice

Stat 406, W1 2020

5 October 2020

1. Suppose $Y_1, \ldots, Y_n$ are iid Normal$(\mu,\ 1)$. Write down the *likelihood* of μ.
2. Suppose $Y_1, \ldots, Y_n$ are *independent* but not identically distributed Normal$(\mu_i,\ 1)$. Write down the *likelihood* of μ.
3. Consider problem 1. What is the expected value of Y_{25}?
4. Consider problem 2. What is the expected value of Y_{25}?
5. Suppose $\widehat{Z}$ is an estimator of ϕ. What is the definition of the *bias* of $\widehat{Z}$?
6. Consider problem 1. What is the *bias* of Y_{25} as an estimator of μ?
7. Consider problem 1. What is the *variance* of Y_{25}?
8. Consider problem 1. What is the *bias* of $\overline{Y}_n = \frac{1}{n}\sum_{i=1}^n Y_i$ as an estimator of μ?
9. Consider problem 1. What is the *variance* of $\overline{Y}_n = \frac{1}{n}\sum_{i=1}^n Y_i$?
10. Consider problem 1. Assume n is even. Is the *variance* of $\widetilde{Y}_n = \frac{1}{n/2}\sum_{i=1}^{n/2} Y_{2i}$ larger or smaller than that of $\overline{Y}_n$? (Could you find the variance if asked?)
11. Consider the previous problem. What is the expected value of $\widetilde{Y}_n$?
12. How does the bias of $\widetilde{Y}_n$ compare to the bias of $\overline{Y}_n$?
13. What is the MSE of $\overline{Y}_n$ *as an estimator of* μ? That is
$$E\left[(\overline{Y}_n - \mu)^2\right] = ???$$
14. What is the MSE of $\overline{Y}_n$ *as an predictor of* Y_{n+1}? That is
$$E\left[(\overline{Y}_n - Y_{n+1})^2\right] = ???$$
15. Suppose $Y_1, \ldots, Y_n$ are independent Normal with means $x_i^\top \beta = \sum_{j=1}^p x_{ij}\beta_j$ and variance σ^2.
    a. Ignoring the trivial cases ($\mathbf{x}_i = \mathbf{x}_j \forall i \neq j$ or $\beta = 0$) are the Y's identically distributed?
    b. What is $E[Y_{25}]$?
    c. What is $E\left[\sum_{i=1}^n Y_i\right]$?
    d. Let $\hat{\beta} = (\mathbf{X}^\top\mathbf{X})^{-1}\mathbf{X}^\top\mathbf{y}$. In this formula, what is random?
    e. What is $E[\hat{\beta}]$?
    f. What is $\mathrm{Var}[\hat{\beta}]$?