STAT 460/560: Statistical Inference 1 **Winter Term 1 (Fall) 2026**

Assignment 1

Due date: Sept. 17, 11:59 pm. *Instructor: Daniel McDonald and Benjamin Bloem-Reddy*

Instructions

Academic integrity policy: I encourage you to discuss verbally with other students about the assignment. However, you **must** write your answers by yourself. If you discuss the assignment with anyone (a classmate or anyone else), you must say so at the top of your solutions. Use of AI tools to produce full or partial answers to these questions or to related questions is strictly prohibited. This includes tools like Co-pilot, auto-complete, etc. Also, refrain from looking at answer keys from other schools, previous years, or the Math Stack Exchange. For more information, see: the syllabus or contact the instructor.

1. Solutions must be **handwritten** (with any computer simulations being the only exception). It must be legible. Difficult-to-read solutions will be marked down or ignored.
2. *Justify your answers formally. (Err on the side of too much detail so that the teaching team knows exactly what you're doing and why.)*
3. Submit your work by 11:59 pm (Vancouver time) on the due date to Canvas (.pdf).
4. For questions that require computer simulations, you may use any language from the list of `(R, Julia, Python)`, with level of support provided by the teaching team decreasing further in the list. Please include in your PDF submission the relevant plots generated by your simulation code, and append your code (in PDF) to your submission. Please retain the code you used, as you may be asked for it if required.

Questions

1. A box contains w white balls and b black balls. A ball is chosen at random. The ball is then replaced, along with d more balls of the same color (as the chosen ball). Then another ball is drawn at random from the box. Show that that probability that the second ball is white does not depend on d.
2. Similar to (Wasserman, 2004) 1.3.

   Show that

   (a) $\limsup_{n\to\infty} A_n = \{x \in S : \text{for all } n \in \mathbb{N} \text{ there exists } N \geq n \text{ such that } x \in A_N\}$.

   (b) $\liminf_{n\to\infty} A_n = \{x \in S : \text{there exists } N \in \mathbb{N} \text{ such that } x \in A_n \text{ for all } n \geq N\}$.

   Comments:

   - This is the reason why the set $\{\limsup_{n\to\infty} A_n\}$ is often denoted $\{x \in S : x \in A_n \text{ infinitely often}\}$ and $\{\liminf_{n\to\infty} A_n\}$ is often denoted $\{x \in S : x \in A_n \text{ eventually}\}$.
   - These sets are useful later on when studying convergence of estimators.

3. (a) Prove that if $A_1, A_2, \ldots$ are mutually exclusive, then $\mathbb{P}(A_n) \to 0$ as $n \to \infty$.
   (b) Suppose I flip a fair coin forever. Let A_n be the event that the n^{th} flip is a head. Since the coin is fair, $\mathbb{P}(A_n) = 1/2$. Notice that $\mathbb{P}(A_n) \not\to 0$ as $n \to \infty$. How, then, can the previous problem still be true? Each A_n refers to a different coin flip, after all. Explain your answer well.
4. Wasserman (2004) 2.7.
5. Prove that for a random variable Z, $P(Z = z) = 0$ if and only if the CDF of Z, $F_Z(z)$, is continuous at z.
6. Let $Y = 3X + 2$ where X has pdf $f_X(x) = \frac{1}{\beta} e^{-x/\beta}$ (it has exponential distribution). Compute $\mathbb{E}[Y]$ four different ways:
   (a) Use the pdf of Y and the definition of $\mathbb{E}[Y]$.
   (b) Use the pdf of X and the definition of $\mathbb{E}[3X + 2]$.
   (c) Use $\mathbb{E}[X]$ and the linearity of expectations.
   (d) Use the moment generating function of Y.
7. Wasserman (2004) 3.4.

References

Larry Wasserman. *All of Statistics*. Springer, New York, 2004.