

Assignment 2

*Due date: Sept. 24, 11:59 pm.**Instructor: Daniel McDonald and Benjamin Bloem-Reddy*

Instructions

Academic integrity policy: I encourage you to discuss verbally with other students about the assignment. However, you **must** write your answers by yourself. If you discuss the assignment with anyone (a classmate or anyone else), you must say so at the top of your solutions. Use of AI tools to produce full or partial answers to these questions or to related questions is strictly prohibited. This includes tools like Co-pilot, auto-complete, etc. Also, refrain from looking at answer keys from other schools, previous years, or the Math Stack Exchange. For more information, see: the syllabus or contact the instructor.

1. Solutions must be **handwritten** (with any computer simulations being the only exception). It must be legible. Difficult-to-read solutions will be marked down or ignored.
2. *Justify your answers formally. (Err on the side of too much detail so that the teaching team knows exactly what you're doing and why.)*
3. Submit your work by 11:59 pm (Vancouver time) on the due date to Canvas (.pdf).
4. For questions that require computer simulations, you may use any language from the list of (R, Julia, Python), with level of support provided by the teaching team decreasing further in the list. Please include in your PDF submission the relevant plots generated by your simulation code, and append your code (in PDF) to your submission. Please retain the code you used, as you may be asked for it if required.

Questions

1. [Wasserman \(2004\)](#) 4.3.
2. [Wasserman \(2004\)](#) 5.16.
3. [van der Vaart \(1998\)](#) 2.6. (Just consider the case of random variables, rather than random vectors.)
4. (a) [van der Vaart \(1998\)](#) 2.10.
(b) Do the same for $o_{\mathbb{P}}(1)O_{\mathbb{P}}(1) = o_{\mathbb{P}}(1)$.
(c) Explain carefully why $o_{\mathbb{P}}(1)+O_{\mathbb{P}}(1) \neq o_{\mathbb{P}}(1)$: give a counterexample, or show where your argument for (a) fails.
5. [van der Vaart \(1998\)](#) 2.11. (This problem highlights a subtle defect in the stochastic o-notation, and is something to keep in mind going forward.)
6. Let X be a non-negative random variable. Show that if, for some $p > 0$, $\mathbb{E}[X^{-p}] < \infty$, then $\mathbb{P}(X = 0) = 0$. (Hence, the distribution of X does not have an atom at 0.) Does the Gamma distribution have an atom at zero when its parameters are positive?

References

A. W. van der Vaart. *Asymptotic Statistics*. Cambridge Series in Statistical and Probabilistic Mathematics. Cambridge University Press, Cambridge, 1998.

Larry Wasserman. *All of Statistics*. Springer, New York, 2004.