STAT 460/560: Statistical Inference 1 **Winter Term 1 (Fall) 2026**

Assignment 3

Due date: Oct. 1, 11 p.m. *Instructor: Daniel McDonald and Benjamin Bloem-Reddy*

Instructions

Academic integrity policy: I encourage you to discuss verbally with other students about the assignment. However, you **must** write your answers by yourself. If you discuss the assignment with anyone (a classmate or anyone else), you must say so at the top of your solutions. Use of AI tools to produce full or partial answers to these questions or to related questions is strictly prohibited. This includes tools like Co-pilot, auto-complete, etc. Also, refrain from looking at answer keys from other schools, previous years, or the Math Stack Exchange. For more information, see: the syllabus or contact the instructor.

1. Solutions may be handwritten or typeset with LaTeX (see the course website for a template). It must be legible. Difficult-to-read solutions will be marked down or ignored.
2. *Justify your answers formally. (Err on the side of too much detail so that the teaching team knows exactly what you're doing and why.)*
3. Submit your work by 11:59 pm (Vancouver time) on the due date to Canvas (.pdf).
4. For questions that require computer simulations, you may use any language from the list of `(R, Julia, Python)`, with level of support provided by the teaching team decreasing further in the list. Please include in your PDF submission the relevant plots generated by your simulation code, and append your code (in PDF) to your submission. Please retain the code you used, as you may be asked for it if required.

Questions

1. Show that continuity is important for the Continuous Mapping Theorem. That is, construct a sequence X_n that converges in probability to a random variable X and a function ϕ is not continuous. Show that $\phi(X_n) \not\to \phi(X)$ in probability.
2. Wasserman (2004) 5.13.
3. Let $X_1, X_2, \ldots$ be i.i.d. $\mathsf{Bern}(p)$ random variables and let $\overline{X}_n = n^{-1}\sum_{i=1}^n X_i$.
   (a) Show that $\sqrt{n}(\overline{X}_n - p) \rightsquigarrow \mathsf{N}(0,\ p(1-p))$
   (b) Show that for $p \neq 1/2$, $\overline{X}_n(1-\overline{X}_n)$ (an estimator of the variance) satisfies
   $$\sqrt{n}\big(\overline{X}_n(1-\overline{X}_n) - p(1-p)\big) \rightsquigarrow \mathsf{N}(0,\ (1-2p)^2 p(1-p)).$$
   (c) Suppose Y_n is a sequence of random variables that satisfies $\sqrt{n}(Y_n - \theta_0) \rightsquigarrow \mathsf{N}(0,\ \sigma^2)$ for some values θ_0 and σ^2. Let $\phi : \mathbb{R} \to \mathbb{R}$ such that $\phi'(\theta_0) = 0$ but $\phi''(\theta_0)$ exists and is not equal to 0. In

this setting, the **second-order Delta Method** gives

$$n(\phi(Y_n) - \phi(\theta_0)) \rightsquigarrow \frac{\sigma^2 \phi''(\theta_0)}{2} \chi_1^2.$$

where χ_1^2 is the χ-squared distribution with 1 degree of freedom.
Show that for $p = 1/2$, $n\big(\overline{X}_n(1 - \overline{X}_n) - 1/4\big) \rightsquigarrow \frac{1}{4}\chi_1^2$.

4. Let $X_1, \ldots, X_n$ be i.i.d. $\mathsf{Unif}(0, \theta)$ random variables. Consider the estimator $\hat{\theta}_n = 2\overline{X}_n$ of θ.

   (a) Find the bias and variance of $\hat{\theta}_n$.

   (b) Find the mean squared error of $\hat{\theta}_n$.

   (c) Is $\hat{\theta}_n$ a consistent estimator of θ? Justify your answer.

   (d) Find the asymptotic distribution of $\hat{\theta}_n$.

5. Let $X_1, \ldots, X_n$ be i.i.d. $\mathsf{Unif}(0, \theta)$ random variables. Consider the estimator $\hat{\theta}_n = \max\{X_1, \ldots, X_n\}$ of θ.

   (a) Find the bias and variance of $\hat{\theta}_n$.

   (b) Find the mean squared error of $\hat{\theta}_n$.

   (c) Is $\hat{\theta}_n$ a consistent estimator of θ? Justify your answer.

   (d) Find the asymptotic distribution of $\hat{\theta}_n$.

6. Wasserman (2004) 7.6.